
Recent Advances in Particle Swarm Optimization for Large Scale Problems

Danping Yan

College of Public Administration,
Huazhong University of Science and Technology,
Wuhan, 430074, China

Yongzhong Lu*

School of Software Engineering,
Huazhong University of Science and Technology,
Wuhan, 430074, China
Fax: +86 27 87792251
E-mail: hotmailuser@163.com

*Corresponding author

Min Zhou

College of Public Administration,
Huazhong University of Science and Technology,
Wuhan, 430074, China

Shiping Chen

Data61,
Commonwealth Scientific and Industrial Research Organization,
Marsfield, 2122, Australia

David Levy

School of Electrical and Information Engineering,
University of Sydney,
Sydney, 2006, Australia

Jicheng You

School of Software Engineering,
Huazhong University of Science and Technology,
Wuhan, 430074, China

Abstract:

Accompanied by the advent of current big data ages, the scales of real world optimization problems with many decisive design variables are becoming much larger. Up to date, how to develop new optimization algorithms for these large scale problems and how to expand the scalability of existing optimization algorithms have posed further challenges in the domain of bio-inspired computation. So addressing these complex large scale problems to produce truly useful results is one of the presently hottest topics. As a branch of the swarm intelligence based algorithms, particle swarm optimization (PSO) for coping with large scale problems and its expansively diverse applications have been in rapid development over the last decade years. This review paper mainly presents its recent achievements and trends, and also highlights the existing unsolved challenging problems and key issues with a huge impact in order to encourage further more research in both large scale PSO theories and their applications in the forthcoming years.

Keywords: swarm intelligence; particle swarm optimization; large scale optimization problem; cooperative coevolution; ensemble evolution; static grouping method; dynamic grouping method.

Biographical notes: Danping Yan received her PhD in Economics at the Huazhong University of Science and Technology (HUST), China. She is currently an associate professor at the College of Public Administration, HUST. Her research interests are related to public economics and policy, bio-inspired optimization computation, intelligent information processing, smart city. She has published many research papers at national and international journals and conference proceedings.

Yongzhong Lu received his PhD in Mechanical Electrical Engineering at the HUST, China. He is currently an associate professor at the School of Software Engineering, HUST. His research interests include issues related to bio-inspired optimization computation, software engineering, pattern recognition and image processing. He is the author of many research papers published at national and international journals and conference proceedings.

Min Zhou received his PhD in Natural Geography in Peking University, China. He is currently a lecturer at the College of Public Administration, HUST, China. His research interests are related to natural geography, environment research and management and intelligent information processing. He has published many research papers at national and international journals and conference proceedings.

Shiping Chen is a principal research scientist at CSIRO Data61. He also holds an adjunct associate professor title with the University of Sydney through teaching and supervising PhD/Master students. He has been working on distributed systems for over 20 years with focus on performance and security. He has published over 100 research papers in these research areas. He is actively involved in computing research community through publications, journal editorships and conference PC services, including WWW, EDOC, ICSOC and IEEE ICWS/SCC/CLOUD. His current research interests include secure data storage & sharing and secure multi-party collaboration. He is a senior member of the IEEE.

David Levy is a professor at the School of Electrical and Information Engineering, University of Sydney, Australia. His research interests are related to software engineering and its applications. He has published a good number of research papers at national and international journals, conference proceedings as well as chapters of books.

Jicheng You is currently pursuing the M. S. degree at the School of Software Engineering with the HUST of China. His research includes computational intelligence and software engineering.

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1 Introduction

With the emergence of big data era, quantity real world problems, such as gene recognition in bio-informatics, inverse problems of chemical kinetics and biological systems, vehicle routing in traffic networks, electronic system design, resource scheduling problem, connection weight optimization in training deep neural network architectures, and so on, are becoming more complex. It is difficult for researchers and practitioners to deal with analysis, capture, data curation, search, sharing, storage, transfer, visualization, querying and information privacy by using originally traditional data processing approaches. Likewise, increased landscape complexity, characteristic alteration and exponentially expanded search space also pose new challenges in the domain of bio-inspired techniques. How to expand these bio-inspired optimization algorithms has become a well recognized challenging expectation task.

Some representative exemplars among these bio-inspired algorithms cover evolutionary algorithm, simulated annealing, differential evolution, PSO and ant colony optimization. They often suffer from the curse of dimensionality when they are applied to solve large scale optimization problems. Accordingly, overcoming these existing difficulties has received an increasing attention from academic communities. In recent years, many valuable attempts have been motivated and proposed. Most noticeable contributions have been reported in the leading journals and the international conferences and workshops. Particularly speaking, PSO has been playing a rather important role among these contributions. This paper aims to highlight the latest development of PSO and its variants to tackle large scale optimization problems and further inspire more future research in both large scale PSO theories and their applications.

The keys to solve large scale optimization problems for the canonical PSO and its variants are to reduce the dimensionality of particle swarm data sets and to improve their diversity. In the light of reducing dimensionality status, the PSO approaches to cope with large scale optimization problems may be classified into two categories: dimensional reduction based cooperative coevolution approaches and non-dimensional reduction based ensemble evolution approaches. In the former, most researchers often take diverse divide-and-conquer measures to decompose large scale particle swarm data sets into relatively lower dimensional data subsets, and concurrently consider their coevolution phenomena to smooth away large scale optimization problems. Different decomposition styles lead to twofold methods, namely, static and dynamic grouping methods. In the latter, most researchers have either proposed particularly biological mutation, selection and crossover mechanisms, or brought forward neighborhood topologies, or employed effective optimization operators in the course of swarm evolution throughout the whole search

space. Actually, these methods without divide-and-conquer strategies focus on individual competition and population cooperation to run specific dynamic trajectories regardless of dimensionality reduction. Obviously, the distinct critical discrepancy between dimensional reduction based cooperative coevolution approaches and non-dimensional reduction based ensemble evolution approaches is whether to adopt divide-and-conquer strategies or not during the process of tackling large scale optimization problems. The hierarchical taxonomy of the PSO solutions to dealing with large scale optimization problems is consolidated in Figure 1. Besides, the corresponding summaries of major PSO variants of the non-dimensional reduction based ensemble evolution and dimensional reduction based cooperative coevolution methods are shown in Tables 1 and 2, respectively.

The remainder of the paper is organized as follows: Section 2 gives a brief description of the canonical PSO for large scale optimization problems. Section 3 depicts the recent achievements of large scale PSO. Section 4 presents the recent trends and challenges of large scale PSO and finally comes to conclusions and gives suggestions on future research work of large scale PSO theories and applications.

2 Description and formulation of canonical PSO for large scale optimization problems

Addressing large scale problems is attributed to the following definition:

$$\min/\max F(\vec{x}) = f(x_1, x_2, \dots, x_n), n \geq 100, \vec{x} \in X \subseteq R^n, (1)$$

where $X \subseteq R^n$ denotes the decisive space with n dimensions, $\vec{x} = (x_1, x_2, \dots, x_n)$ represents the decisive variable vector, $f: X \rightarrow R$ stands for a real continuous nonlinear objective function for mapping from n dimensional space to one dimensional fitness value $F(\vec{x})$, and n is the number of decisive variables in large scale problems addressed here.

PSO is considered as one representative and widely used swarm intelligence paradigm proposed by Kennedy and Eberhart in 1995 for solving optimization problems. Due to its easy implementation and effectiveness, PSO has so far greatly progressed and been successfully used in solving large scale optimization problems. It tactfully replies on the flocking behavior synergy of flying birds on the way to the specific destination to locate the local and global optima through the whole search space. Each particle with a position and a velocity flying in an n dimensional search space can be depicted by the following iterative equations (2)-(3):

$$\begin{aligned} \vec{v}_i(t+1) = \omega \vec{v}_i(t) + c_1 \vec{R}_1(t)(p\vec{best}_i(t) - \vec{x}_i(t)) \\ + c_2 \vec{R}_2(t)(g\vec{best}_i(t) - \vec{x}_i(t)), \end{aligned} \quad (2)$$

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1), \quad (3)$$

Table 1 A summary of major PSO variants of the non-dimensional reduction based ensemble evolution methods.

Method	Author	Algorithm abbreviation	Dimensionality	Brief comment
Social learning methods	Wang et al. (2011)	GOPSO	[30, 100]	Employs generalized opposition based learning and Cauchy mutation
	Montes de Oca et al. (2010)	IPSO	[100, 100]	Incremental optimizer with a growing population size and local search
	Montes de Oca et al. (2011)	IPSOLS	[50, 1000]	Combines components of social and individual learning mechanisms
	Cheng and Jin (2015)	SL-PSO	[100, 1000]	Adopts better learning and dimensional dependent parameter control
Biological mechanism based methods	Hsieh et al. (2008)	EPUS-PSO	[100, 1000]	A optimizer with an efficient population utilization strategy
	Ali (2010)	SAPSO	[100, 1000]	Both safety distance and proximity index are introduced
	García-Nieto and Alba (2011)	RPSO-vm	[50, 1000]	A velocity modulation and a restarting mechanisms are introduced
	Wang et al. (2013)	DNPSO	[30, 100]	Employs diversity enhancement and neighborhood search strategies
	Cheng and Jin (2015)	CPSO	[100, 5000]	A pairwise competition mechanism is introduced
	Rather et al. (2015)	BPSO	[110, 470]	A mixed integer dynamic PSO based method
	Hou et al. (2015)	ESEPSO	[80, 80]	A canonical PSO algorithm
	Cai et al. (2015)	GDPSO	[100, 11000]	A greedy discrete PSO framework
	Banka and Dara (2015)	HDBPSO	[2000, 7000]	A Hamming distance based binary PSO algorithm
	Zhang et al. (2015)	MR-IDPSO	[50, 10000]	A MapReduce based improved discrete PSO algorithm
	Chen et al. (2015)	AMSPSO	[80, 230]	A PSO with adaptive multi-swarm strategy
	Cheng et al. (2015)	BGPSO	[33, 69]	An efficient binary PSO algorithm
	Sun et al. (2016)	PSO	[3840, 3840]	A canonical PSO algorithm
	Zhang et al. (2016)	DCSO	[100, 1000]	A dynamic competitive swarm optimizer based on population entropy
Neighborhood topology based methods	Fan et al. (2014)	FT-DNPSO	[30, 1000]	Dynamic neighborhood based on kernel fuzzy clustering
Local search based methods	Zhao et al. (2008)	DMS-PSO	[100, 1000]	A dynamic multi-swarm particle swarm optimizer
	Zhao et al. (2010)	DMS-PSO-SHS	[1000, 1000]	hybridize dynamic multi-swarm and a sub-regional harmony search
Sampling based methods	Chu et al. (2011)	BHPSO	[10, 100]	Consider the aspects of the three boundary handling techniques
	Engelbrecht (2011)	HPSO	[10, 1000]	A heterogeneous PSO
	Budhrajia et al. (2013)	RoEPSO	[10, 30]	Guided re-initialization schema into PSO
	Sahu et al. (2015)	DPSO	[64, 128]	An improved discrete multiple PSO based strategy
	Van Zyl and Engelbrecht (2015)	PSO-IS	[500, 2000]	A novel strategy of particle swarm initialization
	Van Zyl and Engelbrecht (2016)	PSO-VS	[500, 2000]	Group-based stochastic scalability of PSO velocities
Hybrid cooperation based methods	Chu et al. (2008)	FBSA	[10, 30]	A novel fast bacterial swarming algorithm
	Zhang and Yi (2011)	SFIPSO	[10, 30]	A scale free fully informed PSO algorithm
	Lin et al. (2014)	GA-PSO	[13, 13]	A hybrid both genetic algorithm and PSO algorithm
	Aziz and Tayarani-N (2014)	AMPSO	[100, 500]	Uses a population based optimizer with a multiple local search procedure
	Tang et al. (2014)	IQPSO	[40, 1000]	An improved quantum behaved PSO algorithm
	Li et al. (2015)	PS-ABC	[60, 500]	Combines local search with global search in artificial bee colony
	Gong et al. (2016)	DPSO	[06, 30]	A discrete PSO algorithm
	Ouyang et al. (2016)	HHSPSO-GDS	[30, 30]	A hybrid harmony search PSO with global dimension selection

Table 2 A summary of major PSO variants of the dimensional reduction based cooperative coevolution methods.

Method	Author	Algorithm abbreviation	Dimensionality	Brief comment
Static grouping methods	Van den Bergh and Engelbrecht (2004)	CPSO	[10, 30]	Cooperative particle swarm optimizer-SK(HK)
	Jiao et al. (2011)	CCPSO	[10, 10]	Cooperative particle swarm optimizer based on catastrophe
	Jiang and Wang (2014)	BPSO	[02, 60]	Bare-bone PSO algorithm
Random dynamic grouping methods	Li and Yao (2009)	CCPSO	[100, 1000]	Incorporates the random grouping and the adaptive weighting schemes
	Li and Yao (2012)	CCPSO2	[100, 2000]	Adopts a new PSO position updating rule
Learning dynamic grouping methods	Sun et al. (2012)	CPSO-SL	[20, 1000]	A CPSO algorithm with statistical variable interdependence learning
	Ismail and Engelbrecht (2012)	CPSO	[05, 30]	A measurement of diversity for the cooperative optimizer
	Lee et al. (2015)	DCCPSO	[30, 90]	Dynamic cooperatively coevolving PSO

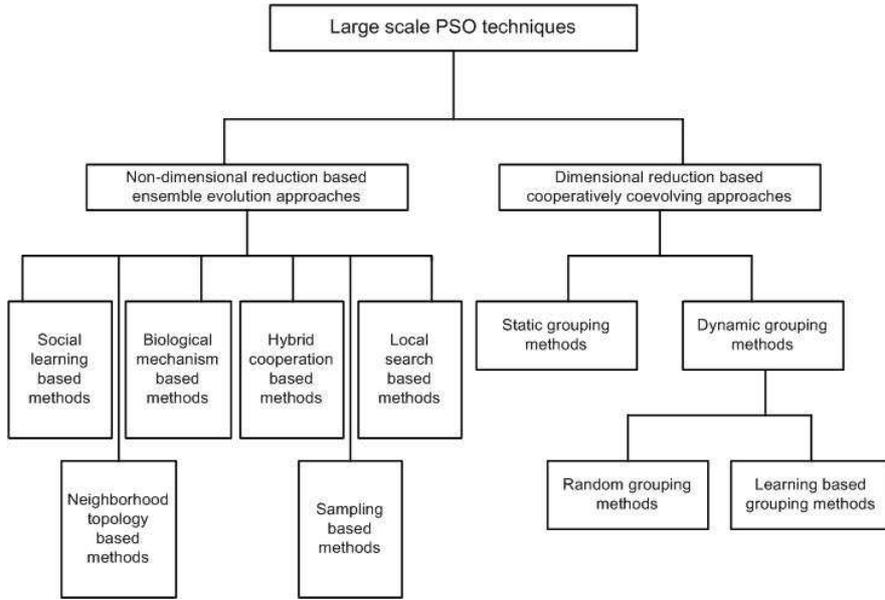


Figure 1 The hierarchical taxonomy of the PSO solutions to dealing with large scale optimization problems.

where t is the iterative variable, $\vec{v}_i(t)$ and $\vec{x}_i(t)$ are the velocity and the position of the i -th particle, ω is the termed inertia weight, $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are two positive random vectors with $[0, 1]^n$, $p\vec{best}_i(t)$ and $g\vec{best}_i(t)$ are so far the best local and global solutions of the i -th particle, $c_1R_1(t)(p\vec{best}_i(t) - \vec{x}_i(t))$ and $c_2R_2(t)(g\vec{best}_i(t) - \vec{x}_i(t))$ are referred as the cognitive component and the social component.

The equations (1)-(3) are composed of the canonical PSO for large scale optimization problems. During the process of solving large scale optimization problems, various dimensional reduction and diversity enhancement strategies are needed to put into effect.

Compared with other population-based metaheuristic algorithms such as evolutionary algorithm, genetic algorithm, differential evolution, ant colony optimization, artificial bee colony, and so on, PSO similarly encounters the deficient curse of dimensionality when solving large scale optimization problems. This means that as the dimensional size of the tackled problem increases, the PSO's performance obviously becomes deteriorated because the search space is exponentially enlarged and the landscape complexity and characteristic alteration are elevated. As is known to all, either evolutionary algorithm or genetic algorithm is inspired by biological evolution, such as reproduction, mutation, recombination and selection, and has crossover and mutation operators. Differential evolution optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality, and also has crossover operator. Ant colony optimization is based on probabilistic pheromone densities to solve combinatorial optimization problems which can be boiled down to finding good paths through graphs. Artificial bee colony is based on the intelligent

foraging behavior of honey bee swarm (employed bees, onlookers and scouts). However, the idea of PSO is to mimic the behaviors of flying birds in the sky. It has three special indispensable components as mentioned above in the equation (2) and better performance of parallel computing. Moreover, it has less complex computation and faster convergence and is easily understood and programmed. Accordingly, most of the dimensional reduction and non-dimensional reduction techniques can be widely embedded in the canonical PSO and its variants and their applications. As for these aspects, other population-based metaheuristic algorithms are not comparable to PSO. Empirically, from the present statistical references with regard to population-based metaheuristic algorithms for solving large scale optimization problems in Tables 1 and 2, the quantity of references of PSO is more than that of any other population-based metaheuristic algorithm while the quantity of references of artificial bee colony is the smallest. In addition, in terms of the reference quantities, other population-based metaheuristic algorithms are ranked by the descending sequence below: differential evolution, evolutionary algorithm + genetic algorithm, ant colony optimization.

3 Recent achievements in large scale PSO

3.1 Dimensional reduction based cooperative coevolution approaches

It is known that there are much more contributions to treating large scale optimization problems effectively. Among the solutions, the classical attempts to overcome these intractable matters are to lower the high dimensionality. Generally, these attempts are called the

divide and conquer approaches or the dimensional reduction based cooperative coevolution approaches. In accordance with the decisive variable grouping or partitioned styles, the dimensional reduction based cooperative coevolution approaches can be parted into two general subcategories: static and dynamic grouping methods. Likewise, different dynamic grouping strategies also result in two subcategories: random or learning based dynamic grouping methods. The recent advances of both static grouping methods and dynamic grouping methods concentrate on algorithmic development and applications of large scale PSO.

3.1.1 Static grouping methods

While static grouping methods are applied to a wide range of large scale optimization problems, they keep the grouping factors constant all through the evolutionary course.

- Van den Bergh and Engelbrecht (2004) presented two variants of the original PSO algorithm which were called the cooperative particle swarm optimizer-SK and cooperative particle swarm optimizer-HK), employing cooperative behaviors to significantly improve the performance of the original PSO algorithm. These are achieved by using multiple swarms to optimize different components of the solution vectors cooperatively. Main framework of the cooperative particle swarm optimizer-SK is based on the original decomposition method with a special difference defined by Potter and De Jong in 1994 where a vector was partitioned into k s -dimensional subproblems and $n = k \times s$. A concatenation of all global best particles from all k swarms is called a context vector \hat{y} which is used to compute the fitness of a particle in a swarm. In the cooperative particle swarm optimizer-HK, the original PSO algorithm and the cooperative particle swarm optimizer-SK are incorporated so that the cooperative particle swarm optimizer-SK is performed for one cycle, followed by the original PSO algorithm in the next cycle. A simple information exchange method between the cooperative particle swarm optimizer-SK and the original PSO algorithm is conducted as the context vector \hat{y} in the cooperative particle swarm optimizer-SK after one iteration is applied to replace a randomly selected particle in the original PSO algorithm. If a new global best particle is discovered in the original PSO algorithm, this vector will be replaced by a randomly selected particle in the cooperative particle swarm optimizer-SK section. Applications of the new PSO variants on several benchmark optimization problems show a marked improvement in performance over the original PSO algorithm.

Afterwards, Jiao et al. (2011) adopted a cooperative particle swarm optimizer based on catastrophe to address the flow shop production scheduling problem with uncertainties in modern manufacturing environments, where the fuzzy processing time is considered, the duration time of intermediate is unlimited, and the maximum membership function of mean value has been applied to solve the nonlinear fuzzy scheduling model, in order to convert the fuzzy optimization problem to the general optimization problem.

Then, based on the cooperative coevolution framework, Jiang and Wang (2014) decomposed the original high dimensional clustering problem to several subproblems, each of which is evolved by an optimizer called the bare-bone PSO algorithm independently. In addition, they also designed a new centroid-based encoding schema for each particle and applied the Chernoff bounds to decide a proper population size.

3.1.2 Dynamic grouping methods

Different from static grouping methods, dynamic grouping methods have dynamic grouping factors when they are executed throughout the search space. Currently, dynamic grouping factors encompass both random grouping factor and learning based grouping factor. They both are very helpful to improve the diversity of the particle swarm in handling large scale optimization problems.

- Random grouping methods

In the study of the random grouping methods, Li and Yao (2009) presented a cooperative coevolving PSO (CCPSO) algorithm incorporating the random grouping and the adaptive weighting schemes, which have been shown to be effective for handling high dimensional nonseparable problems. The proposed CCPSO algorithm outperforms a previously developed coevolving PSO algorithm on nonseparable functions of 30 dimensions. Furthermore, the scalability of the proposed algorithm to high dimensional nonseparable problems of up to 1000 dimensions is examined.

Subsequently, Li and Yao (2012) presented a new CCPSO2 algorithm in an attempt to address the issue of scaling up PSO variants in solving large scale optimization problems up to 2000 real valued variables. The proposed CCPSO2 builds on the success of an early CCPSO algorithm which employs an effective variable random grouping technique, and adopts a new PSO position updating rule that relies on Cauchy and Gaussian distributions to sample new points in the search space, and a scheme to dynamically determine the coevolving subcomponent sizes

of the variables. On high dimensional problems ranging from 100 to 2000 variables, CCPSO2 performs significantly better than a state-of-the-art evolutionary algorithm sep-CMA-ES, two existing PSO algorithms, and a cooperative coevolving differential evolution algorithm when solving large scale and complex multi-modal optimization problems.

- Learning based grouping methods

Besides the random grouping strategies, the learning based grouping methods focus on the acquired learning experiences either before or during the evolutionary process. These experiences are obtained for the intrinsic characteristics of large scale optimization problems and appropriately determine the emerging prior probabilities of the interacting better decisive variables in the subgroups in an endeavor to enhance the diversity of the particle swarm.

Decomposition decision regarding variable interdependencies plays a significant role in the algorithm's performance. Considering the algorithm's variable interdependencies, Sun et al. (2012) proposed a CPSO algorithm with statistical variable interdependence learning (CPSO-SL). A statistical model is proposed to explore the interdependencies among variables. With these interdependencies, the algorithm partitions large scale problems into overlapping small scale subproblems. Moreover, a CPSO framework is proposed to optimize the subproblems cooperatively. Simultaneously, theoretical analysis is also presented for further understanding of the proposed CPSO-SL. The success of the cooperative particle swarm optimizer has been ascribed to its increased diversity.

Ismail and Engelbrecht (2012) proposed a measurement of diversity for the cooperative particle swarm optimizer which is compared with three other diversity measures to establish the most appropriate diversity measure for the cooperative particle swarm optimizer.

Lee et al. (2015) introduced a dynamic cooperatively coevolving PSO into distributed multi-robot predictive control model to guarantee asymptotic stability of the multi-agent systems whose state vectors are coupled and nonseparable in a cost function. The proposed algorithm is proposed to deal with the formation control problem. As the proposed algorithm finds the Nash equilibrium state in a distributed way, robots can quickly move into a desired formation from their locations.

3.1.3 Discussion

Dimensional reduction based cooperative coevolution approaches generally include three classical steps as follows: problem decomposition, subcomponent optimization and cooperative combination. The above-mentioned static grouping methods perform better on separable problems. Note that the non-separable or epistasis problems are referred to the problems with strong interacting variables, i.e., the influence of each variable on the fitness value is dependent of any other variables. Since the static grouping methods are inefficient in handling non-separable or epistasis problems, the dynamic grouping methods have been proposed to deal with them. They dynamically change the grouping structure so that they can easily detect variable interactions and assign interacting variables to the same subcomponent, while the fixed subcomponent sizes in the static grouping methods remain unchanged in the same subcomponents over the optimization process. Compared with the static grouping methods, the random grouping methods show a comparably efficient performance on scalable non-separable or epistasis problems. However, their performances become ineffective when the number of interacting variables grows. As a result, the dynamic grouping methods are proposed to increase the placing chances of the interacting variables in the same subcomponent especially when the number of interacting variables becomes very large. They usually rely on the prior knowledge of the problem characteristics either before or during optimization process to decide the appropriate grouping of the interacting variables. Accordingly, the dynamic grouping methods have more advantages than the random grouping methods when solving high dimensional problems with complex variable interactions.

3.2 Non-dimensional reduction based ensemble evolution approaches

More generally, non-dimensional reduction based ensemble evolution approaches focus on the introduction of specially innovative evolutionary operators such as biological mutation, selection and crossover, neighborhood topology, local search, sampling, social learning, population alteration and hybrid cooperation, and so forth during the exploration and exploitation search process. These operators are not the same as ones used in low dimensional optimization problems.

3.2.1 Biological mechanism based methods

- Hsieh et al. (2008) presented a particle swarm optimizer with an efficient population utilization strategy. Variable particles are used to remove the redundant particles so as to enhance the search ability. Moreover, an moving vector is added

in the the velocity equation and proportionally varies by a certain probability to another particle's local best position. A search range sharing strategy is also constructed to keep the particles from falling into the local minima and to make the global minimum found easily.

Ali (2010) investigated a novel PSO algorithm, where an adaptive inertia weight is presented at three evolution levels. The most important features presented are both the safety distance introduced to move the particle through its current position, and the proximity index.

García-Nieto and Alba (2011) incorporated two new mechanisms into the PSO algorithm with the aim of enhancing its scalability. A velocity modulation method is first applied in the movement of particles in order to guide them within the region of interest. Then a restarting mechanism avoids the early convergence and redirects the particles to the promising areas in the search space.

Wang et al. (2013) proposed a hybrid PSO algorithm, which employs a diversity enhancement mechanism and a neighborhood search strategy to achieve a trade-off between exploration and exploitation abilities.

In Cheng and Jin (2015), a novel competitive swarm optimizer for large scale optimization is proposed. A pairwise competition mechanism is introduced, where the particle that loses the competition will update its position by learning from the winner. A theoretical proof of convergence is provided, together with empirical analysis of its exploration and exploitation abilities, showing that the proposed algorithm achieves a good balance between exploration and exploitation and effectively solves problems of dimensionality up to 5000.

Rather et al. (2015) proposed a mixed integer dynamic PSO based method for optimal dynamic reactive power allocation in large scale wind integrated power systems.

In Hou et al. (2015), a mathematical model which includes the variation of both wind direction and wake deficit, is proposed. The problem is formulated by using leveled production cost as the objective function. The optimization procedure is performed by a PSO algorithm with the purpose of maximizing the energy yields while minimizing the total investment.

In Cai et al. (2015), a greedy discrete PSO framework for large scale social network clustering is suggested. The particle statuses are redefined under a discrete scenario. The status updating rules are reconsidered based on the network topology. A greedy strategy is designed to drive particles to a promising region.

A Hamming distance based binary PSO algorithm is used to handle the feature selection, classification and validation of gene expression data. Hamming distance is introduced as a proximity measure to update the velocity of particles in binary PSO framework to select the important feature subsets in Banka and Dara (2015).

Zhang et al. (2015) proposed a MapReduce based improved discrete PSO algorithm to select the optimal composited service from thousands of functionally equivalent services with different quality of service.

In Chen et al. (2015), a PSO with adaptive multi-swarm strategy is proposed to solve the capacitated vehicle routing problem with pickups and deliveries, which includes goods delivery/pickup optimization, vehicle number optimization, routing path optimization and transportation cost minimization. The proposed PSO employs multiple PSO algorithms and an adaptive algorithm with punishment mechanism to search the optimal solution, which can deal with large scale optimization problems.

Cheng et al. (2015) presented an efficient binary PSO algorithm for loss minimization in distribution network reconfiguration.

For the interferences suppression of a large scale antenna array after a sub-array configuration, a generalized sidelobe canceller weighting approximation algorithm is presented. The irregular sub-array configuration is obtained by PSO in Sun et al. (2016).

In Zhang et al. (2016), a dynamic competitive swarm optimizer based on population entropy is proposed. The new algorithm is derived from the competitive swarm optimizer. The new algorithm uses population entropy to make a quantitative description about the diversity of population, and to divide the population into two subgroups dynamically. During the early stage of the execution process, to speed up convergence of the algorithm, the subgroup with better fitness will have a small size, and worse large subgroup will learn from the small one. During the late stage of the execution process, to keep the diversity of the population, the subgroup with better fitness will have a large size, and small worse subgroup will learn from the large group.

3.2.2 Neighborhood topology based methods

- A PSO variant approach with dynamic neighborhood that is based on kernel fuzzy clustering and variable trust region methods, is proposed, where a cooperative coevolution

strategy incorporated with a kernel fuzzy C-means clustering strategy is introduced to divide the high dimensional problems into the subproblem spaces. Simultaneously, an independent variable ranges change adaptably by using the variable trust region learning method so as to expedite the convergence process. Additionally, a dynamic neighborhood topology assists this PSO variant to keep away from premature convergence in Fan et al. (2014).

3.2.3 Local search based methods

- In Zhao et al. (2008), a dynamic multi-swarm particle swarm optimizer is presented. In the optimizer, the whole population is divided into a large number of sub-swarms which are regrouped frequently by using various regrouping schedules and are exchanged among the particles in the whole swarm. In the meanwhile, the Quasi-Newton method is employed to improve its local search ability.

Successively in Zhao et al. (2010), the dynamic multi-swarm particle swarm optimizer and a sub-regional harmony search are hybridized to form another PSO variant. A modified multi-trajectory search algorithm is also applied frequently on several selected solutions.

3.2.4 Sampling based methods

- Chu et al. (2011) analyzed and compared the aspects of the three boundary handling techniques, namely, the random, absorbing and reflecting schemes in the high dimensional complex problems, and they indicated the insights and the specific information about the performance of PSO.

Engelbrecht (2011) developed a heterogeneous PSO which allows particles to randomly select a different behavior at each iteration from a behavior pool. This algorithm is significantly more scalable than a selection of homogeneous PSO algorithms on large dimensional instances of the benchmark functions.

Budhரா et al. (2013) introduced a guided re-initialization schema into PSO to increase the diversity portrayed by particles. The algorithm implements a form of teleportation by which particles are randomly re-initialized in the search space once their behavior becomes predictable. The predictability is modeled using a hypersphere of variable radius, centered at the best known solution.

Sahu et al. (2015) presented an improved discrete multiple PSO based strategy to map applications

on both 2-D and 3-D mesh connected networks on chip. A part of the initial population in the multiple PSO approach is deterministically generated.

In Van Zyl and Engelbrecht (2015), a novel strategy of particle swarm initialization particularly for high dimensional problems is proposed. The initialization strategy encourages the swarm to focus on exploitation rather than exploration, thereby allowing it to find fairly good solutions, even in the face of high dimensionality and very large search spaces.

Van Zyl and Engelbrecht (2016) conducted an analysis of the group-based stochastic scalability of PSO velocities on high dimensional problems.

3.2.5 Social learning methods

- Wang et al. (2011) presented an enhanced PSO algorithm, which employs generalized opposition based learning and Cauchy mutation to overcome the problem of premature convergence.

Montes de Oca et al. (2010) presented two PSO algorithms: a) the incremental particle swarm optimizer, which is a PSO algorithm with a growing population size in which the initial position of new particles is biased toward the best so far solution, and b) the incremental particle swarm optimizer with local search.

Montes de Oca et al. (2011) presented a case study of a tuning in the loop approach for redesigning a PSO algorithm for tackling large scale continuous optimization problems. An incremental social learning framework which combines components of social and individual learning to increase learning rate, is introduced in a PSO variant. In the course of redesigning the PSO variant, the tuning in the loop approach is used to proceed within six phases, namely, selection of a local search method, alteration of calling and controlling the local search method, using vectorial PSO rules, penalizing bound constraints violation, and fighting stagnation with restarting.

Cheng and Jin (2015) introduced social learning mechanisms into PSO to develop a social learning PSO. Unlike classical PSO variants where the particles are updated based on historical information, including the best solution found by the whole swarm and the best solution found by each particle, each particle in the proposed PSO learns from any better particles in the current swarm. In addition, to ease the burden of parameter settings, the proposed PSO adopts a dimensional dependent parameter control method.

3.2.6 Hybrid cooperation based methods

- A novel fast bacterial swarming algorithm for high dimensional function optimization is presented in Chu et al. (2008). The proposed algorithm combines the foraging mechanism of E-coli bacterium introduced in bacterial foraging algorithm with the swarming pattern of birds in block introduced in PSO. It incorporates the merits of the two bio-inspired algorithms to improve the convergence for high dimensional function optimization. A new parameter called attraction factor is introduced to adjust the bacterial trajectory according to the location of the best bacterium. An adaptive step length is adopted to improve the local search ability.

Zhang and Yi (2011) proposed a scale free fully informed PSO algorithm. In the proposed algorithm, a modified Barabási-Albert model is used as a self-organizing construction mechanism, in order to adaptively generate the population topology exhibiting scale free property. The swarm population is divided into two sub-populations: the active particles and the inactive particles. The active particles fly around the solution space to find the global optima, while the inactive particles are iteratively activated by the active particles via attaching to them, according to their own degrees, fitness values, and spatial positions. Therefore, the topology will be gradually generated as the construction process and the optimization process progress synchronously. Moreover, the cognitive effect and the social effect on the variance of a particle's velocity vector are distributed by its contextual fitness value, and the social effect is further distributed via a time-varying weighted fully informed mechanism.

In Lin et al. (2014), a hybrid of both genetic algorithm and PSO algorithm is proposed. Genetic algorithm aims to cover every region of the search space while PSO searches the neighborhood to further prune the good solutions. This proposed approach is used for high dimensional extensive subspace clustering because it can improve clustering quality by removing irrelevant and redundant dimensionality in high dimensional clustering problems.

In order to find a near optimal Latin hypercube design, Aziz and Tayarani-N (2014) proposed a new version of PSO algorithm, which uses a population based optimizer as the evolutionary part and a multiple local search procedure as the refinement part of the algorithm. To manage the problem constraints, the proposed algorithm utilizes a ranked order value rule, which converts

the continuous space of solutions into the point permutation space. Furthermore, to maintain the population diversity, the meta-Lamarckian learning strategy is applied to the local search procedure of the algorithm.

Tang et al. (2014) proposed an improved quantum behaved PSO algorithm for continuous nonlinear large scale problems based on memetic algorithm and memory mechanism. The memetic algorithm is used to make each particle gain some experience through a local search before being involved in the evolutionary process, and the memory mechanism is used to introduce a so-called 'bird kingdom' with memory capacity, both of which can improve the global search ability of the algorithm. Each dimension of a particle updates with the same random number. Thus, the convergent speed is increased and the local search ability is enhanced.

Li et al. (2015) proposed a hybrid algorithm, which combines the local search phase in PSO with two global search phases in artificial bee colony for the global optimum, to address high dimensional optimization problems.

Gong et al. (2016) introduced a discrete PSO algorithm for resolving high order graph matching problems, which incorporates several redefined operations, a problem specific initialization method based on heuristic information and a problem specific local search procedure.

Ouyang et al. (2016) presented a hybrid harmony search PSO with global dimension selection for improving the performance of PSO. In the presented PSO, a new global velocity updating strategy is introduced to enhance the neighborhood region search of the current best solution and to get a better trade-off between convergence rate and robustness. Additionally, a dynamic nonlinear decreased inertia weight is utilized to balance the global exploration and local exploitation. Moreover, the improvisation mechanism of harmony search is implanted in the presented algorithm and a global dimension selection is employed in the improvisation process, which can effectively accelerate convergence. Global best information sharing strategy is developed to link the harmony search PSO two layer exploration frames.

3.2.7 Discussion

Non-dimensional reduction based ensemble evolution approaches focus on special divide-and-conquer strategies for high dimensional problems. Among these approaches, biological mechanism based methods, neighborhood topology based methods, local search based methods, sampling based methods, social

learning methods and hybrid cooperation based methods are playing different roles in tackling high dimensional problems. The biological mechanism based methods and the hybrid cooperation based methods gain more prominent popularity. Newly emerging biological evolution strategies are more popular with researchers and are easily implemented in reality, while hybrid evolution algorithms are mutually complementary, collaboratively progressed and comparatively complicated. Although the neighborhood topology based methods focus on the dynamic neighborhood topology to avoid premature convergence, their steps together with other relevant clustering, learning incorporated methods are rather difficult. The local search based methods are tightly related to the neighborhood topology based methods because they usually exploit dynamic and randomized neighborhood topologies to choose the exploitation and exploration conditions so as to improve their local searching ability. The sampling based methods stress heterogeneous data via boundary handling techniques and guided initialization schemas. Unlike classical PSO variants, the social learning methods introduce social learning mechanisms into PSO to develop a social learning PSO. In contrast to individual learning, social learning has the advantage of allowing individuals to learn behaviors from others without incurring the costs of individual trials and errors.

3.3 Empirical analysis and comparisons of different approaches

The typical PSO approaches for large scale problems are reduced to two categories: dimensional reduction based cooperative coevolution approaches and non-dimensional reduction based ensemble evolution approaches. Since the former approaches solve large scale problems through problem decomposition, subcomponent optimization and cooperative combination, they generally consume less computational cost than the latter approaches do. However, the number of the former approaches is less than that of the latter approaches. Among the former approaches, two kinds of more successful random dynamic grouping methods which are CCPSO presented by Li and Yao (2009) and CCPSO2 proposed by Li and Yao (2012), are paid much more attention to. The scalability of CCPSO on high dimensional nonseparable problems may be up to 1000 dimensions. Moreover, CCPSO2 performs significantly better than a state-of-the-art evolutionary algorithm sep-CMA-ES on high dimensional problems ranging from 100 to 2000 variables. Compared with dimensional reduction based cooperative coevolution approaches, non-dimensional reduction based ensemble evolution approaches are dependent on divide-and-conquer strategies for high dimensional problems. They perform comparably better than dimensional reduction based cooperative coevolution approaches on higher

dimensional problems ranging from 2000 to 11000 variables. The biological mechanism based methods are the most successful non-dimensional reduction based ensemble evolution approaches. CPSO proposed by Cheng and Jin (2015) achieves a good balance between exploration and exploitation and effectively solves problems of dimensionality up to 5000. GDPSO suggested by Cai et al. (2015) solves large scale social network clustering problems with 11000 variables. HDBPSOA presented by Banka and Dara (2015) handles the feature selection, classification and validation of gene expression data with 7000 variables. MR-IDPSO selects the optimal composited service from 10000 functionally equivalent services with different quality of service. From these empirical analysis and comparisons, it is suggested that when solving high dimensional problems, we first choose the biological mechanism based methods among non-dimensional reduction based ensemble evolution approaches such as CPSO, HDBPSOA, GDPSO, MR-IDPSO, and so on regardless of their more computational cost consumed. Then we consider the random dynamic grouping methods among dimensional reduction based cooperative coevolution approaches such as CCPSO, CCPSO2, and so forth. In addition, the social learning methods among non-dimensional reduction based ensemble evolution approaches and the learning based grouping methods among dimensional reduction based cooperative coevolution approaches are also taken into consideration because of their distinctly prominent performances on high dimensional problems.

4 Conclusion

4.1 Challenges and trends in large scale PSO

With the rapid development of big data, real world optimization problems with diverse and stringent constraints have become much more complex. As driven by the information technology and social networks, the data volumes of optimization problems are dramatically increasing. Generally speaking, these large scale problems can typically be multi-modal. Accompanied by the increase of the number of decisive variables, the corresponding search solution space varies exponentially. Accordingly, the computational costs in function evaluations are highly expended. Moreover, benchmark problems and performance measures are not yet complete and need to be further developed.

Getting rid of these difficulties has attracted much interest of a good many researchers and practitioners in the domain of bio-inspired computation. How to expand the bio-inspired algorithms to solve large scale optimization problems, how to reduce the dimensionality and how to improve the diversity, are usually and hotly discussed among researchers and practitioners. So far, they have significantly developed a

great number of bio-inspired algorithms to tackle large scale optimization problems. In this paper, we provide a comprehensive review on large scale PSO, which includes description and formulation of canonical PSO for large scale optimization problems, recent achievements of large scale PSO, challenges, trends and future work of large scale PSO. The large scale PSO algorithms are described to present the algorithmic development and their applications. As far as the dimensional reduction based cooperative coevolution approaches are concerned, although static grouping methods present a new cooperative coevolution framework and lay a solid basis of developing a generic divide and conquer methodology, dynamic grouping methods play a more important role in solving real world large scale optimization problems because of their better performances. However, for the non-dimensional reduction based cooperative coevolution approaches, most relevant studies are concentrated on two types of modifications such as defining new biological mutation, selection and crossover and proposing innovative hybrid cooperation during the evolutionary process. In addition, social learning PSO is also worthy of being considered.

4.2 Future work in large scale PSO

Despite the success of large scale PSO in recent years, there exist many unsolved problems which will be more likely to have a huge impact on the further research and progress. We highlight these important matters as follow:

- Obviously, all large scale PSO theories are helpful to gain insight into the evolutionary mechanisms of large scale PSO algorithms. But the theoretical studies about large scale PSO still lag their applications.
- Distribution of particle swarm big data is expected to be investigated so that more intensive big data mining techniques can be adopted to participate in carrying out some true applications like fault detection and filtering as well as image processing.
- Besides, the theoretical analysis of the optimal grouping and its characteristics in the dimensional reduction based cooperative coevolution framework needs consideration.
- Otherwise, large scale PSO benchmark functions should be further balanced and be more similar to the truly real world optimization problems, especially in their dynamic characteristics.
- To the end, it is worth noting that combinatorial problems can be derived from quite different areas and applications in reality and are highly required to be solved by large scale PSO.

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