Review Article

Toward Global Complex Systems Control - The Autonomous Intelligence Challenge

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ABSTRACT

Complex systems are the emerging new scientific frontier with modern technology advance and new parametric domains study in natural systems. An important challenge is, contrary to classical systems studied so far, the great difficulty in predicting their future behaviour from initial time because, by their very structure, interactions strength between system components is shielding completely their specific individual features. Independent of clear existence of strict laws complex systems are obeying like classical systems, it is however possible today to develop methods allowing to handle dynamical properties of such systems and to master their evolution. So the methods should be imperatively adapted to representing system self organization when becoming complex. This rests upon the new paradigm of passing from classical trajectory space to more abstract trajectory manifolds associated to natural system invariants characterizing complex system dynamics. The methods are basically of qualitative nature, independent of system state space dimension and, because of its generic impreciseness, privileging robustness to compensate for not well known system parameters and functional variations. This points toward the importance of control approach for complex system study in adequate function spaces, the more as for industrial applications there is now evidence that transforming a complicated man made system into a complex one is extremely beneficial for overall performance improvement. But this last step requires larger intelligence delegation to the system requiring more autonomy for exploiting its full potential. A well-defined, meaningful and explicit control law should be set by using equivalence classes within which system dynamics are forced to stay, so that a complex system described in very general terms can behave in a prescribed way for fixed system parameters value. Along the line traced by Nature for living creatures, the delegation is expressed at lower level by a change from regular trajectory space control to task space control following system reassessment into its complex stage imposed by the high level of interactions between system constitutive components. Aspects of this situation with coordinated action on both power and information fluxes are handled in a new and explicit control structure derived from application of Fixed Point Theorem which turns out to better perform than (also explicit) extension of Popov criterion to more general nonlinear monotonically upper bounded potentials bounding system dynamics discussed here. An interesting observation is that when correctly amended as proposed here, complex systems are not as commonly believed a counterexample to reductionism so strongly influential in Science with Cartesian method supposedly only valid for complicated systems.

Keywords: Complex Systems; Autonomous Intelligence; System Components; Bounding System Dynamics

1. Introduction

1.1 As observed by paleontologists, human kind very early designed for survival adapted tools extending the action of his hand, a trend since continued up to the extraordinary advance in modern technology during the last decades soon after World War II. Despite drastic change over the centuries, a basic economic competition has been always remaining, inevitably leading to research of higher and more secure performances for overall considered processes. At each step direct action of human operator has been transferred to the tool first, to the machine later. The delegation from human operator to his machine was mainly concerning

efficiency, accuracy, power, and safety, all of technical nature. It is now concerning decision and coordination of action, ie of intelligence, with correlatively some machine autonomy, leaving a supervisory position to operator. After the ascent of industrial civilisation in last two centuries, recent advance of modern today technology has produced base components with unsurpassed performance, implemented in new extended systems. They gather very heterogeneous elements each in charge of a part of system global action. For economic efficiency they are directly integrated in the design, with extremely strong mutual interactions definitely conditioning final system output. Thirty years ago mechanical devices in charge of necessary information flux inside the system have been replaced by electronic more powerful ones, much easier to operate safely in different environments and carrying much more information, giving rise to Mechatronics[1]. In recent years Information Technologies have deeply modified the possibility to connect system elements between themselves and to distant bases for collecting and/or producing needed information for system action. This is summarized on **Figure 1**, where Block 1 is corresponding to the millennium long development of base system, Block 2 describes the structure of controlled system with recent mechatronic components, and Block 3 is representing the present next step toward system independence.

1.2 At early beginning the main concern was improvement of delivered power through management and regulation of its flux inside the system, whereas with last two steps the accent is now on information flux organization unavoidable for both control and autonomous decision. New conditions concerning information handling have to be satisfied for storing and dispatching it with adapted hardware, as well as specific tools for its correct manipulation for the coming step of autonomous intelligent action, whether the systems are networked[2] or not. So new requirements are emerging for necessary improvement of information flux circulation in all aspects related to present step delegation for more autonomy and intelligence into produced artefacts. The delegation could take different aspects and levels[3] up to complete replacement of human intelligence in highest futuristic stage. Here a modest step is considered first as the problem is also concerning system dynamics as well, and more importantly trajectory definition. As explained later, system trajectory is escaping from operator capability, contrary to previous classical case where it is possible to set input system control for following a prescribed trajectory. Observation of living creatures clearly shows the importance of well identified sequences, separable in as many tasks during the development of their current life. Similarly, man-made machine activity can also be split in units corresponding to accomplishment of a specific task, whose combination is often needed for reaching the goal assigned to the machine. As many trajectories are associated to a task, the problem is no longer one-to-one and the difficulty for the system is to define its own trajectory once the elementary task has been assigned. To operate the system in task space instead of classical trajectory space, earlier methods are not fully adapted. On the other hand, in parallel to man made transformation, research has been reaching domains where the behaviour of studied objects is itself strongly depending on interactions between elementary components[4]. The strength of interactions can even completely mask elementary interactions between basic components, and final system behaviour is, due to importance of nonlinearities, generally outside the range of application of classical methods. This is understandable inasmuch as the system often reaches its stage after exhibiting a series of branching along which it was bifurcating toward a new global state the features of which are not usually amenable to a simple local study, being remembered that the branching phenomenon is resting upon a full nonlinear and global behaviour. In the following, some aspects of the problem raised by giving the systems autonomous intelligence will be discussed, and in particular to what extend IT can be useful for this major step.

2. The New Paradigm

2.1 First is the understanding that adding a new, over designed, layer on top of previous ones when using recent technology is not simply possible for system transformation toward more autonomous intelligence. Lower level layers should also be reorganized to fit with the new role they have to play in the presence of branched states. Various attempts have been proposed so far to deal with the handling of new bifurcated phenomena, both in Applied Mathematics and in Control methods. In first class, results[5] on "chaotic" state show that the later represents the general case of nonlinear non integrable systems[6], and that it is reached for high enough value of (nonlinear) coupling parameter. In second class are belonging extensive new control methods often (improperly) called "intelligent"[7], supposed to give systems the ability to behave in a much flexible and appropriate way. However these analyses, aside unsolved stability and robustness problems[8], still postulate that system trajectory can be followed as in classical mechanical case, and be acted upon by appropriate means. In present case on the contrary, the very strong interaction between components in natural systems induces as observed in experiments a wandering of trajectory which becomes indistinguishable from neighbouring ones[9], and only manifolds can be identified and separated[10]. So even if it could be tracked, specific system trajectory cannot be modified by any action at this level because there is no information content from system point of view, as already well known in Thermodynamics[11]. Similar situation occurs in modern technology applications to give now systems the possibility to decide their own trajectory for a fixed task assignment and for which there exists in general many allowed trajectories. In both cases there is a shift to a situation where the mathematical structure generates a manifold instead of a trajectory, now needed for fulfilling technical requirements in task execution under imposed (and often tight) economic constraints. This already corresponds to a very important qualitative jump in the approach of highly nonlinear systems, and requires proper tools for being correctly handled.

Strikingly Nature has been facing this issue a few billion years ago when cells with DNA "memory" molecules have emerged from primitive environment. They exhibit the main features engineers try today to imbed in their own constructions, mainly a very high degree of robustness resulting from massive parallelism and high redundancy. Though extremely difficult to understand, their high degree of accomplishment may provide interesting guidelines for technical present problem. So the consequences of enhanced interaction regime between components in a system as concerns its control have to be clarified. In particular, the control inputs in man-made systems will be considered as elements of a more general control space, which can be defined on a reasonable and useful physical base. Then classical control problem[12] with typical control loops guaranteeing convergence of system output toward a prescribed trajectory fixed elsewhere, shifts to another one where the system, from only task prescription, has to generate its own trajectory in the manifold of realizable ones. A specific type of internal organisation has to be set for this purpose which not only gives the system necessary knowledge of outside world, but also integrates its new features at system level[13]. This means that the new controller should handle the fact that trajectory is not prescribed as before but belongs to a manifold related in a meaningful way to the task.

2.2 In summary, classical control line cannot be continued by adding ingredients extending previous results to new intelligent task control. Another type of demand is emerging when mathematically passing from space time local trajectory control to more global *manifold control*. Some results are today available but the corner stone is to give the system its own intelligence based on its own capacities rather than usual dump of outer operator intelligence into an unfit structure. The next question of selecting appropriate information for task accomplishment and linking it to system dynamics has also to be solved when passing to manifold control by defining "useful information"[14]. A possibility is to mimic natural systems by appropriately linking system degrees of freedom for better functioning, i.e. to make it "complex". Another class of problems is related to manipulation of information flux by itself in relation to the very fast development of systems handling this flux. Overall, a more sophisticated level is now appearing which relates to the shift toward more global

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properties "intelligent" systems should have. At each level of structure development, the system should satisfy specific properties represented in corresponding well defined mathematical terms :

a)-*asymptotic* stability for following imposed command, corresponding to Block 1 in **Figure 1**,

b)-*robustness* for facing adverse environment, corresponding to Block 2,

c)-*determinism* at task level, corresponding to Block 3 for guaranteeing that action is worth doing it.

Prior to development of more powerful hardware components, the problem should be solved by proper embedding into the formalism of recent advances in modern functional analysis methods in order to evaluate the requirements for handling this new paradigm. Point a) corresponds to classical system control[15] typically schematized on **Figure 6a**. For most nonlinear systems it is generally depending on rarely explicit Lyapounov method despite a gigantic effort over more than a century. For point b), typically represented on **Figure 6b**, it will first be shown below that, though it is usually non compatible with point a) in classical approach , first two properties a) and b) can be merged by using manifold control developed in Appendices. Concerning point c), typical system control structure complying with autonomous decision property is displayed on **Figure 6c**. Determinism property specific of necessary information handling at this level will be discussed afterward in the framework of IT with its restrictions, and very difficult questions still under study especially in the domain of embedded autonomous systems which are playing an always larger role in modern human civilisation.
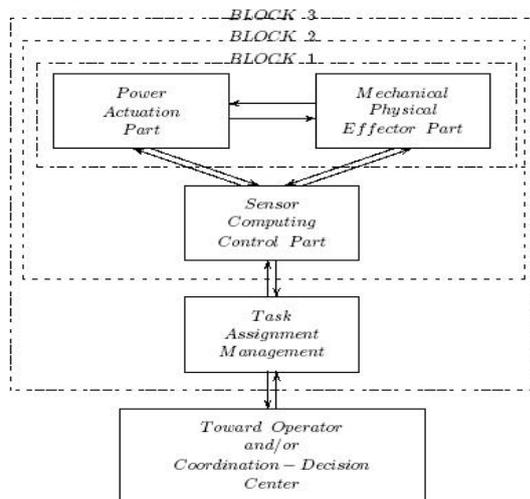

Figure 1. System Structure Evolution with Main

Component Parts :

Block 1 Corresponds to Millennium Long Development of Base System

Block 2 Represents the Structure of Controlled System with Mechatronics Components

Block 3 Includes Present Next Step toward Larger System Independence
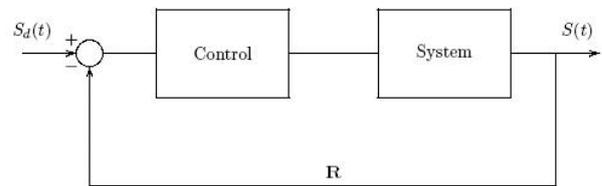

Figure 6a. Simplest Classical Feedback Control Scheme at Block 1 of Figure 1.

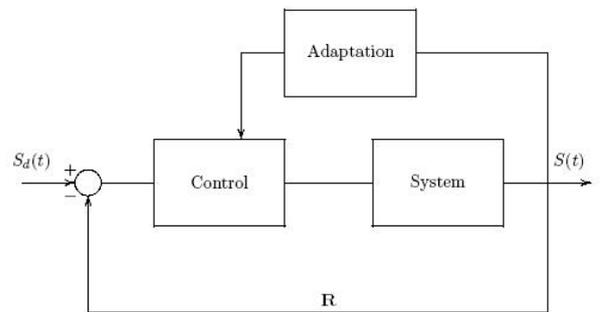A Modifying Box May also Act on Return Loop R

Figure 6b. Classical Improved Feedback Control Loop with Adaptive Box at Block 2 of **Figure 1**
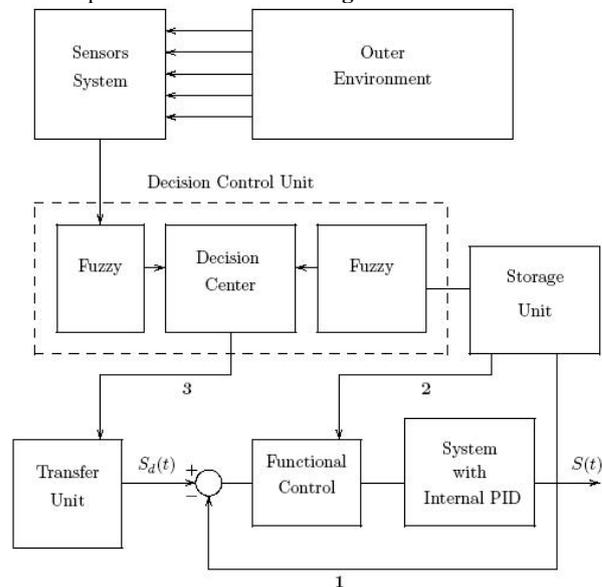

Figure 6c. Intelligent Control Structure at Block 3 of **Figure 1** with Decision Centre (here with Fuzzy type Reasoning) Transferring Order To System (Link 3),

Storage Unit (Usually Neural Type Structure) for Shortening and Backing up Decision (Link 2) and Classical Previous Control Loop (Link 1).

Note that Main Difference with **Figure 6**ab is that Desired Output $S_d(t)$ is now

Internally Fixed by System Itself, and not Given from Outside as Before.

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3. The Approach to Complexity

3.1 To proceed, it is necessary to understand the consequences for control of the wandering nature of system trajectory when becoming complex. As neighbouring trajectories are becoming indistinguishable, the information content in their observation vanishes, so there is no information loss when abandoning their detailed observation. But as the number of degrees of freedom (dof) has not changed (or may become larger at branching), the conclusion is that the system is reaching a new stage where some of the dof are now taken care of by internal system reorganisation, see **Figure 4.** They are moreover in sufficient number so that the only necessary inputs to drive the system are the ones defining the manifold on which system dynamics take place. This self-organisation reducing possible action onto the system from outside is very fundamental[16]. It expresses the simple fact that at higher interactive level between its components, the system can no longer stand with all its dof controlled from outside, but takes itself part of their control in a way which continues its existence. This is not usual textbook case of control with a smaller dimension input control than system dimension. Here because some inputs are just changing from outside to inside the system, they cannot be maintained unless they are in conflict with the new internal organisation fully determining them already. This situation is extremely important as it corresponds to the general case of natural systems, and is accompanying the change into so-called dissipative structure[17].
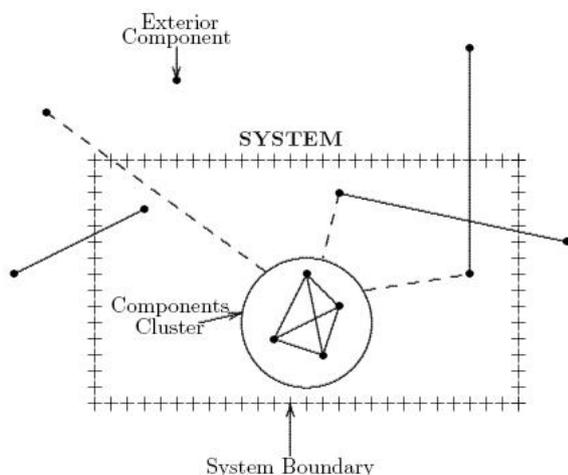

Figure 4. Clustered System Components (in Complex State) are only Globally Interacting

Through their Cluster with Other Interior and Exterior Components (Dashed Links)

The systems in reorganised (and partly self-controlled) situation are *complex ones*[18] (from latin root – *cum plexus*: tied up with), by opposition to simple ones completely controllable from outside. Past some level of *complication* (note its different meaning from complexity), natural systems necessarily become complex once interactions get large enough. The critical threshold for this change can be explicitly stated in terms of system parameters[19], and is shown to correspond to each observed system structure change in as different domains as Physics, Chemistry, Applied Mathematics, Biology, Sociology and Economy[20]. So determining this natural exchange makes it possible to take advantage of complex structure by accepting the *compromise* to reduce the outer control action to the only dof left free after system reorganisation, as there is no information loss in the process which leads to hierarchized system structure[21]. On the other hand, technical systems with large number of dof have to be constructed for accomplishment of complex enough tasks, and evidently the resulting high degree of complication makes the control of such systems very fragile, if not strictly impossible when interactions are becoming large enough for some actual operating parameters under the pressure of "economic" constraints. In mathematical terms, the mixed system representation with a random background noise generally used to mock up this situation does no longer apply[22], see **Figure 5.** It usually corresponds to simple stability result strongly depending on maximum acceptable gain by system actuators and here over passed, calling for another more dynamically detailed representation[23]. The situation described above provides the possibility for man-made systems to exist in a useful and manageable way, with a different approach to system control mainly based on a compromise to avoid too severe constraints on system dynamics, as this is the only workable way for maintaining system existence. Here design parameters, independently fixed in previous scheme, are now determined by the final pre-assigned behaviour when including internal loops corresponding to dof linkages making the system complex. So there is no clear distinction between design and control parameters. This global approach, often

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called "optimal design", should be developed in adequate functional frame as only manifolds, and not trajectories, are now accessible[24]. On the other hand it guarantees locally asymptotic stability of trajectories within a robustness ball the size of which is fixed by the parameters of an equivalence class including the system at hand as an element.
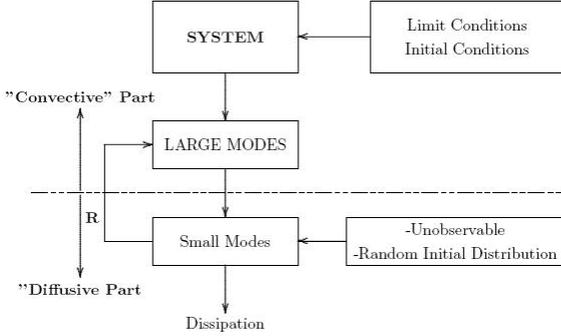

Figure 5. Schematic Block Structure of Complex Type System with Convective and Diffusive Parts and Feedback Link R between them often Mocked up by Averaged Transports Evaluated by Supposing Complete Randomness of Small Modes for all Times (Chandrasekhar Hypothesis)

3.2 The next step is to find the link at system level between task definition and the manifold of (self-organised) trajectories the system can only follow. For living organisms, this property is hardwired into the brain representation of the environment and of the living being itself from experience through their sensory and nervous systems. On the other hand, they have broad range sensors and they filter for each task the relevant information (called ''useful'' here) needed to guide their trajectory. These two properties will be reproduced at system level by first defining a functional of system trajectories expressed in terms of the only invariants characterising the manifold on which they are lying, and by constructing a functional control law which only acts at manifold level. This is possible by considering now a complete trajectory as a ''point'' on a function manifold $x = x(.) \in \mathcal{E}$, rather than usual succession of positions $x(t_j)$ for each time t_j . In this view, stability is obtained as the belonging of $x(.)$ to a pre-selected function space S expressed through a fixed point condition in this space, see **Figure 7.** Then derivatives of the functional with respect to task parameters give system sensitivity to the task and provide the filter matrix selecting the relevant manifold for task accomplishment by the system. General expressions are found by application of Fixed Point Theorem[25] which

can be shown to contain as applications most published results since Lyapounov and Poincaré pioneering work[26], see Appendix **A**. From these elements, the control structure can be constructed with its various parts, see Appendix **B**. For constant bounds, another control law can be found in explicit analytical form by extension of Yakubovic-Kalman-Popov (YKP) criterion[27] to general non-decreasing more accurate bound[28], see Appendix **C**. Both laws give explicit asymptotic stability limit and define the robustness equivalence class within which the property holds, and as expectable the first law is shown to include the second one which may nevertheless be interesting as it has a different expression.
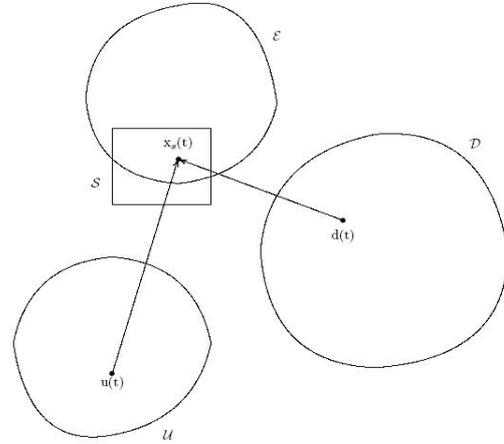

Figure 7. Generating Scheme of Solution $x_S(t, x_S(0),u(t),d(t))$ from Knowledge of Control Functions u(t) $\in$ U and Disturbances d(t) $\in$ D. Determination of Subspaces D' $\subseteq$ D, U' $\subseteq$ U so that $x_S(t) \in$ S is Done by Application of Fixed Point Theorem in Adequate Function Spaces

4. The New System Structure

4.1 Following the scheme developed above, the task controller structure corresponding to Block 3 in **Figure 1** to be developed now comprises the information filter selecting the task relevant information from system sensors, see **Figure 2.** The basic postulate is here that the knowledge of both information and power fluxes at each instant completely determines a system[29]. To escape from "inanimate" world where information flux is rigidly linked to power flux by laws of Physics, the way followed by Nature at early beginning of Earth existence has been to split information and power fluxes by creation of "memory" molecules storing an independent information content, out of which existence of living creatures able to perform their own tasks has been made possible. In present case also, the system should have the

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possibility to escape from previous situation by modifying its behaviour for adapting to adverse effects, whether of internal or external origin, opposing to task accomplishment. So even if usual system control at level of Block 2 proves with robustness to be usually efficient for restoring prescribed trajectory[30], further increase of system performance requires a new degree of autonomy for building its own strategy. An apparently simple possibility would be to give human operator the required information for efficient system drive[31]. However, due to ergonomic constraints and to various limits on technical response times, this action has a limited potential as it does not properly delegate enough freedom to adequate system level for full exploitation of its technical capabilities. Consequently the problem is not only to give the system proper information collected from sensors now utilized in various displays by human operator, but more importantly to put them in a format compatible with system own dynamics. Comparison between human eye, extremely useful to human operator due to very remarkable brain organization, and machines with camera and recognition software stresses the importance to verify that delivered information matches with system possibility of action. Though not possible at general goal level, the only necessary step is to manage system organization so that delivered information meets with each specific task sub-level. In present case this rests upon the concept of "useful" information $\mathcal{J}$[32] to be internally filtered by the system for retaining the only elements relevant for actual action, see **Figure 2**. As stressed above, internal system dynamical effects cannot be distinguished between one another in general case. Using their observation to improve system dynamical control is not possible, in the same way as observing individual molecule motion in a fluid would not improve its global control. So increasing the amount of information from sensors as commonly developed is not the solution. Only relevant information has to be collected, and this justifies why raw sensor information has to be filtered so that only useful information for desired task accomplishment is selected. This is precisely the remarkable capability of living systems to have evolved their internal structure so that this property is harmoniously embedded at each level of organisation corresponding to each level of their

development. Mathematically, usefulness of information rests upon calculating ''utility'' u of events for task accomplishment given by absolute value of logarithmic derivative of Lyapunov function $\mathcal{L}(\mathcal{J})$ selecting trajectories $\mathcal{T}$ of manifold $\mathcal{M}(\mathcal{J})$ inside allowable workspace domain $\mathcal{D}$ meeting task requirement. When expressed in terms of problem parameters and system invariants of trajectories, it automatically selects as most useful the elements against which the system exhibits larger sensitivity. As indicated later, sensitivity threshold has to be fixed so that resulting uncertainty ball is *inside* attractor ball corresponding to asymptotic stability of system trajectories. Elimination of elements to which the system is insensitive prior to any calculation in control unit enormously reduces computation load. In all cases useful information is completely explicit once the system is fixed. When adverse effects are acting, their interaction with the system are changing its dynamical equations and the domain $\mathcal{D}(\mathcal{J})$ is no longer fully available. The remaining domain $\mathcal{D}_c(\mathcal{J})$ is determined by a new Lyapunov function $\mathcal{L}_c$ with the controller guaranteeing asymptotic stability of new system trajectories, from which a new utility function u_c and the new constrained useful information $\mathcal{J}_c$ can be explicitly written. Useful information is thus a class property characterizing from their invariants a family of possible trajectories in workspace which have their representation determined from dynamic and/or general geometric properties.
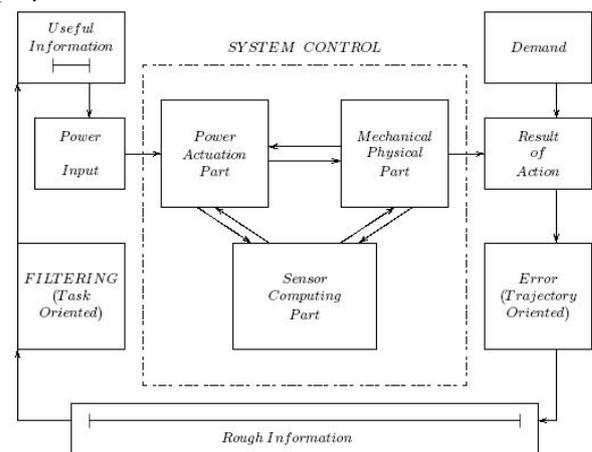

Figure 2. System Structure with Main Components Parts and Information Filtering for Task Orientation

4.2 Because of 1)-operation at task level and 2)-trajectory non distinguishability in general complex

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system, a functional control is defined guaranteeing the trajectory to belong to a selected functional space corresponding to researched properties. For finite dimensional nonlinear and time dependent systems explicit expressions for controller $\mathcal{C}_F$ are obtained in terms of system global characteristics, and exhibit robust asymptotic stability inside the desired function space under mild conditions by application of Fixed Point Theorem[33]. Specific exponential convergence decay[34] and extension to unknown systems[35] are more generally possible. The controller $\mathcal{C}_F$

$$u(t) = -Ke + \Delta u \qquad (1)$$

is the sum of a linear PD type part and an ad-hoc nonlinear part depending on the distance of actual trajectory to allowed manifold trajectories $\mathcal{M}(\mathcal{T})$, and where e is the trajectory error with respect to nominal one belonging to $\mathcal{M}(\mathcal{T})$. The role of additional Δu is to counteract the effect of nonlinear and disturbed remaining parts in system equations, and to define an asymptotic stability ball within which all system trajectories are guaranteed to stay inside selected manifold $\mathcal{M}(\mathcal{T})$. This is very clear on the expression of upper bound on derivative of adapted Lyapunov function along system trajectories by the sum of an attractive spring resulting from controller action and usually repulsive force representing system effect[36]. The controller is functionally robust as it only implies a global bounding function of nonlinear terms, which means that all systems with same bounding function will be asymptotically controlled by $\mathcal{C}_F$. With this controller $\mathcal{C}_F$ working at trajectory level $\mathcal{T}$ it is possible to design the block diagram of task oriented control displayed on **Figure 3.** Independent of lower level controllers inside local subsystem such as actuator and effector boxes to be tuned aside, it mainly implies, on top of the functional controller loop (1) guaranteeing required trajectory following, a second higher level loop (2) of decisional nature based on information $\mathcal{I}_c$ which verifies that the system is actually following a trajectory belonging to the manifold $\mathcal{M}(\mathcal{T})$ corresponding to the assigned task, and opens a search toward this class when conditions are changing. The search can be conducted by interpolation within a preloaded neural network covering the different possible situations over the workspace. In present scheme actual trajectory

followed by the system is not necessarily unique, as there is in general a class of allowed trajectories associated to a task, provided it belongs to asymptotic robustness ball for this (explicit) trajectory controller. So controlled system dynamics are defining a trajectory which is followed until it would escape without controller from acceptable manifold corresponding to the defined task and selected by useful information $\mathcal{I}_{ce}$. A constraint may be further added to limit or to minimize useful information $\mathcal{I}_{ce}$ if required, but the correlative restriction of acceptable trajectory manifold occurs at the expense of system adaptation, so a smooth constraint is often more appropriate if any.

The important point is that the various elements in the chart of **Figure 3** have been constructed so that 1)-they are explicitly expressible in terms of system parameters (or can be reconstituted if unknown) and 2)-they are linked together in completely coherent way through consideration of trajectory as a whole appearing as the good "unit", both qualities needed to create adequate link between the two loops for transferring decisional power, ie "intelligence", to the system. In this sense the system is given its own task consciousness as it does not obey here a strict step by step outer command.
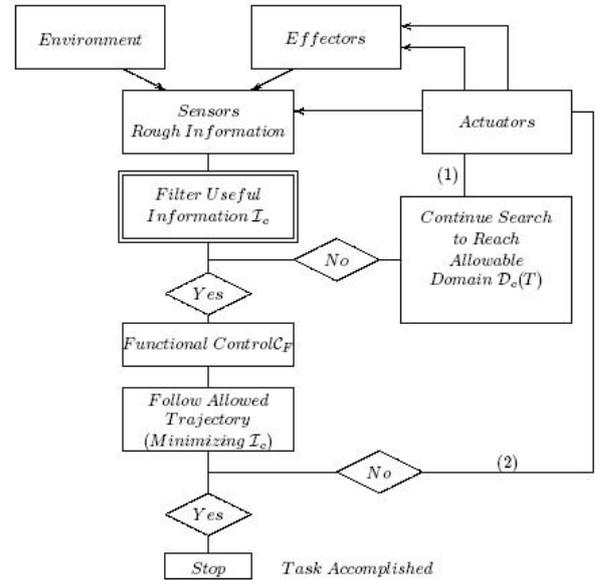

Figure 3. Structure of Filtering Block of **Figure 2** with Task Oriented Control

5. Overall System Determinism

5.1 Intelligence transfer required for system autonomy rests here upon observation that for qualitative improvement of system behaviour it is necessary to give

the system broader information on itself and its environment for better definition of its "next" step. There are two main aspects in this statement. One is dealing with (usually) long run system structure transformation for better adaptation to its task, as for living species during their evolution, and considered today for man made systems such as factories[37]. The other is more modestly in system organization for immediate action, as it is discussed here. So in the threefold system representation of **Figure 1**, the sensor-computing-control part is now taking the important share beyond "Mechatronics" step, by larger role of communication network carrying information inside and outside the system when linked to other ones in networked mode[38]. Handling information flux takes two aspects related to safety. One is concerning task fulfilment in a time compatible with global goal achievement, especially with real time embedded systems, the other with problems related to uncertain system action impact on environment and risk evaluation demanding more secure answer from measurements. Evidently, there is no sense that using their new emerging collective skill, adults in animal colony are only bringing back the food for their off-springs so late that they all die by starvation in the meantime. Same thing occurs in man made systems in appropriate terms to give the system adequate deterministic property, by guaranteeing the (maximum) time delay between information input and action output. For real time effecting system, this represents ability to take advantage of input information before obsolescence and/or mis-conduction, and to drive fast enough system dynamics compared to natural system characteristic response time. The condition is usually analyzed on system flow chart representing data and information exchanges in between system components according to required sub-tasks and taking account of priority and queueing[39]. When developed on classical threefold system representation, there results an evaluation of global system reaction time in terms of components characteristic times usually satisfied by adapted design of the later, owing to the breath of available manufactured components on the marketplace. To illustrate the method developed in the paper, a fully autonomous robot has been built up, see **Figure 8.** The robot is assigned to realize in a finite fixed time interval T, and at a

prescribed place, the task of constructing an object from components dispersed on the floor of a initially defined $\{L \times l\}$ working area, with controller split organization presented here and following "natural" optimal design adapted to task requirements[40], in particular, with carefully chosen size. As obtained from preliminary analysis it can perform its task in a time T_{op} function of system parameters, which for specific values, is much shorter than with usual robots assigned to similar task, showing the potential interest of the method as it will be discussed elsewhere.
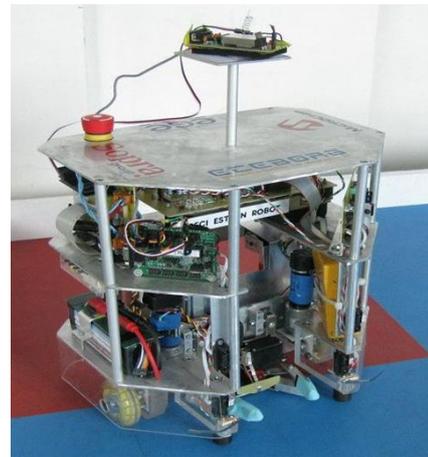

Figure 8. Front Picture of the Test Robot

5.2 More generally, the situation is tenser when many systems are networked because of their possible split response time, leading to very hard and unsolved resources allocation problems[41], and is worse with limited resources in autonomous embedded systems like modern cars produced in automotive industry. In completely integrated network of acting systems, the nature of interactions makes the resulting cluster a complex system again. Analysis of this type of problem is still in infancy even if some typical network schemes have been investigated[42]. With the merging of computers, IT and telecommunications networks, robotics, distributed systems software and multi organizational applications of hybrid technology, the distinction between computers and effecting technical systems (eg robots, grids, sensors,..) becomes somewhat arbitrary. In a way similar to energy distribution[43], the difficulty to distribute intelligence in the network for better efficiency is still open as it may be more convenient to conceive a principal intelligence with dispersed sensors and effectors, each with subsidiary

intelligence (as for instance robotics-enhanced computer system), or conversely, it looks more realistic to think in terms of multiple devices, each with appropriate sensory, processing, and motor capabilities subjected to some form of coordination (an integrated multi-robot system). The key difference is in the complexity and persistence achieved by artefact behaviour, independent of human involvement. Going further, the focus will be on the future generation of technologies in which computers and networks are integrated into everyday environment, rendering accessible a multitude of services and applications through easy-to-use interfaces. Such a vision of "ambient intelligence"[44] will place the user - the individual - at the centre of future developments for an inclusive knowledge-based society for all. In parallel, the huge industrial production increase and the multiplication of production centres is opening the questions of their interaction with environment and of the resulting risk, both domains in which scientific response has been rather modest up to now due to inadequacy of classical "hard" methods to integrate properly their global (and essential) aspect.

6. Conclusion

In today industry the demand for higher performances under economic and environmental constraints cannot be satisfied by simple upgrade of previous components. New phenomena related to handling systems heterogeneity and number of components have recently opened a broad domain of investigations on phenomena related to this new structure. As largely documented, natural systems mostly belong to the broad class of complex structures when interaction between system components becomes strong enough with internal self-organisation minimizing system dependence with respect to outside world. System trajectory then becomes more erratic in state space, and so cannot be distinguished from neighbouring ones. Only manifolds corresponding to system invariants can be separated in general, indicating that the number of outer control inputs is reduced as the system is taking the other control inputs under their own dynamics. Such a compromise is a natural trend expressed by the general principle of autonomy of complex systems, which states that they naturally evolve as a dissipative structure

toward the state giving them the least dependence on outside world compatible with boundary conditions. This is culminating with living systems which, because the previous compromise forces them to maintain a metabolism driving them to a finite life, have extended survival principle to whole species by reproduction. In this sense, and in agreement with Aristoteles view, existence of complex systems is the first necessary step from natural background structure toward independence and isolation of a domain which could later manage its own evolution by accessing to life and finally to thought. In the same way as Nature has been a few billion years ago able to cross complexity barrier by inventing "memory" molecules able to store information required for living creatures development, there is today a similar problem to design complex systems only able to accept intelligence delegation for successful operation, as it is not worth to develop individual highly performing components without taking advantage of their capability at global system level. Because both power *and* information fluxes are now concerned, different problems are identified concerning internal system coordination and control, information flux handling and communication between a networked cluster of systems. Analysis of passage to complex stage shows that previous steps defined for simpler system situation have to be reassessed for meeting the new requirements imposed by complex status. In particular for power flux it is mandatory that asymptotic stability be satisfied inside a robustness ball of at least the size of system uncertainty. So, following bottom-up approach described here, classical trajectory system control should be upgraded to more adapted task control. The construction of new controller is made possible in two steps by developing an explicit trajectory control of functional nature, which is asymptotically stable and robust enough to cover the manifold of possible trajectories. Second, by introducing the concept of ''useful'' information, a task functional expressed in terms of system parameters is set up which defines compatible trajectory manifold. From them a double loop is written giving the system the possibility to accomplish the task for any allowed trajectory by determining its path from its own elements. In this sense it has gained more independent behaviour and, similar to very advanced living systems, is able to

operate autonomously at its own level. The next important step concerning mainly information flux is to guarantee determinism expressing system ability to perform its task in acceptable time compatible with system goal. This problem is solved for internal system structure by flow chart analysis, but is still unapproachable when dealing with clusters implying collaborative networked systems, ie precisely when the network becomes complex from information flux side, in contrast with complexity from power flux side discussed above.

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APPENDIX A

The problem of complex system dynamics can be reformulated in the following way. Consider the finite dimensional nonlinear and time dependent systems

$$\frac{dx_s}{dt} = F_s(x_s(t), u(t), d(t), t)$$

(A1)

where $F_s(.,.,.,.) : \mathbf{R}^n \times \mathbf{R}^m \times \mathbf{R}^p \times \mathbf{R}^1_+ \rightarrow \mathbf{R}^n$ is a $\mathbf{C}^1$ function of its first two arguments, $x_s(t)$ the system state, $u(t)$ the control input, and $d(t)$ the disturbance acting upon the system. In full generality the control input $u(t)$ can be either a parameter which can be manipulated by operator action in man made system or more generally an acting parameter on the system from its environment to the variation of which it is intended to study the sensitivity. To proceed, this equation will be considered as a generic one with now $u(.) \in \mathcal{U}$ and $d(.) \in \mathcal{D}$, where $\mathcal{U}$ and $\mathcal{D}$ are two function spaces to be defined in compatibility with the problem, for instance $\mathcal{L}^p$, $\mathcal{W}_n^p$, $\mathcal{M}_n^p$, respectively Lebesgue, Sobolev and Marcenkievitch − Besicovitch spaces[45] related to useful and global physical properties such as energy and/or power boundedness and

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smoothness. Now for $u(.)$ and $d(.)$ in their definition spaces, eqn(A1) produces a solution $x_s(.)$ which generates a manifold $\mathcal{E}$ and the problem is now to analyze the partitioning of $\mathcal{U}$ and $\mathcal{D}$ corresponding to the different (normed) spaces S within which $x_S(t, x_S(0), u(t), d(t)) \in \mathcal{E}$ can be embedded, see **Figure 7.** When S is $\mathcal{M}_n^2$ for instance, simple stability property is immediately recovered. The base method to express this property is the use of Fixed Point Theorem in its various representations[25]. The generality of this approach stems to the fact that all stability and embedding methods written so far since pioneering work of Lyapounov and Poincaré[26] are alternate expressions of this base property[23]. The problem is easily formulated when there exists a functional bounding $F_s(.,.,.,.)$ in norm. For instance, a usual bound (related to Caratheodory condition in Thermodynamics) is in the form of generalized Lipschitz inequality[46]

$$|F_s(x,u,d,t) - F_s(x',u',d',t)|$$
$$\leq L_s(t)|x - x'|^{\beta_s} + L_1(t)(|u - u'|_\infty + L_2(t)|d - d'|_\infty)$$
(A2)

Then by substitution one gets for $x_s(t)$ the bound

$$|x_s(t)| \leq |x_s(t_0)| + \int_{t_0}^{t}\left[L_s(t')|x_s(t')|^{\beta_s} + R_1(t')\right]dt'$$
(A3)

with $R_1(t) = L_1(t)|u(t)|_\infty + L_2(t)|d(t)|_\infty$ and when solving for $x_s(t)$

$$\int_{t_0}^{t} R_1(t')dt' \geq |x_s(t)| \,_2F_1(1,\beta_s^{-1},1+\beta_s^{-1}, -Sup[L_s(t)/R_1(t)]|x_s(t)|^{\beta_s})$$
(A4)

with $_2F_1(\alpha,\beta,\gamma,z)$ the Gauss hypergeometric function. So there is a fixed point $x_s(t) \in \mathcal{L}_\infty$ for $u,d \in \mathcal{L}_\infty$ exhibiting simple stability property. The result extends to more general non decreasing bounding function $g(|x - x'|,.)$ instead of polynomial one in eqn(A2)[46]. It should be observed that the obtained global BIBO type bound found here is sensitive to the way the integral is performed with respect to upper bounding of the various terms in the RHS of eqn(A1). With respect to this rough property, the problem at hand is now to research if there exists a controller which guarantees robust stability in as large as possible ball around the origin. The main remark is that to make the researched jump from BIBO result for

the class of equations considered with appropriate time dependent bounding functions, only finite power input is required which is technically doable and justifies the analysis.

If the function $F_s(., ., ., t)$ is not well known due to un-modeled internal system phenomena, only a model

$$dx_m/dt = F_m(x_m(t), u(t), 0, t)$$
(A5)

of system dynamics can be used, with solution $x_m(t,t_0, x_{0m}, u)$ and initial condition $x_{0m} = x_m(t_0)$ simply supposed to belong to $\mathbf{R}_n$. This constraint may also be relaxed in specific cases which will not be discussed here. It is reasonable to suppose that the "distance" between $F_s(., ., .)$ and $F_m(., ., .)$ is not too large, and more precisely that there exists $K, L \in \mathbf{R}_1^+$ so that similar (Lipschitz type) generalized inequalities[46]

$$|F_s(x, u, d, t) - F_m(x, u, d, t)| \leq N_1(t)g_{ms}(|x|) + N_2(t)|u|_\infty + N_3(t)|d|_\infty$$
(A6)

$$|F_m(x, u, d, t) - F_m(x', u', d', t)| \leq L_m(t)g_m(|x - x'|) + L_2(t)|u - u'|_\infty + L_3(t)|d - d'|_\infty$$
(A7)

hold on top of eqn(3) for $(x, u, d), (x', u', d') \in \Omega_0 \times U \times D$, bounded positive time dependent functions $N_j(t)$, $L_j(t)$, $L_m(t)$, and with $\Omega_0 \in \mathbf{R}_n$ the domain where the solution of eqns(A1) will exist when $x_s(t_0), x_m(t_0) \in \mathbf{B}_\rho$, the ball centered at the origin with radius $\rho \in \mathbf{R}_1^+$. Using eqns(A6,A7) one gets

$$d|d_{ms}(t)|/dt \leq L_m(t)g_m(|d_{ms}(t)|) + \Theta(t)$$
(A8)

with $d_{ms} = x_m - x_s$ and

$$\Theta(t) = N_1(t)g_{ms}(|x_s(t)|) + (N_2(t) + L_2(t))|u|_\infty + (N_3(t) + L_3(t))|d(t)|_\infty$$
(A9)

When $x_s(t)$ is stable, $\Theta(t)$ is bounded and eqn(A8) is majorized by

$$d|d_{ms}(t)|/dt \leq g_m(|d_{ms}(t)|) + Sup_t\Theta(t)L_m(t)$$
(A10)

Using generalized Bellman-Gronwall inequality, it follows that

$$|d_{ms}(t)| \leq G_{ms}^{-1}[G_{ms}(d_{ms}(0)) + \int_0^t L_m(t')dt']$$
(A11)

where $G_{ms}(x) = \int^x [g_{ms}(\xi) + Sup_t(\Theta(t)/L_m(t))]^{-1} d\xi$,

showing that $|d_{ms}(t)|$ is in general at most bounded. Model system (*m*) is in a bounded neighborhood of real system (*s*). If (*m*) belongs to robustness ball of (*s*) it may have same asymptotic properties. Determination of this property in each specific case is thus very important, such as for instance when Caratheodory condition is satisfied[46]. Extension to the case where the system is unknown and the model cannot be set down is possible with projective methods[35,47].

APPENDIX B

To get the analytical form of the controller c_F, a further step is obtained when representing eqn(A1) close to a solution $x_{s0}(t)$ obtained for a specific input $u_0(t)$ by eqn(A3) with eqn(A4) gives for error *e* the form

$$\frac{de}{dt} = A(t)e + B_0(t)\Delta u + G_s(e, x_s, \Delta u, d, t)$$

(B1)

when splitting the linear part with $x_s(t) = x_{s0}(t) + e(t)$ and

$$u(t) = u_0(t) + Ke + \Delta u$$

(B2)

It will be supposed that, as very often from physical reasons, the nonlinear part satisfies the bounding inequality

$$\|G_s(e, x_s, \Delta u, d, t)\| \leq M_s(t)g_s(\|e(t)\|, t)$$

(B3)

and that the linear gain *K* in eqn(4) is such that the linear part of the system is asymptotically stable (for time independent matrices A and B_0 this means that $A = A_0 - B_0K$ is Hurwitz). Supposing there exists positive definite matrices *P* and *Q* such that $PA + A^TP + dP/dt = -Q$, one can define the (positive definite) Lyapunov function

$$L(x_s) = <ePe>$$

(B4)

using the bra ket formalism. Its derivative along system trajectories is given by

$$d L(e)/dt = -<eQe> + <ePB\Delta u> + <ePG_s>$$

(B5)

Chosing from eqn(B3) the controller form

$$\Delta u = -\beta M_s g_s \frac{B^T Pe}{\|Pe\| + \varepsilon f}$$

(B6)

where β and ε are two parameters and $f = f(.)$ is the driving function to be defined later, eqn(A4) transforms after substitution into the inequality

$$d\mathcal{L}(e)/dt \ \leq \ -<eQe> + \varepsilon M_s g_s f + M_s g_s$$
$$\frac{\|Pe\|^2 - \beta\|B^T Pe\|^2}{\|Pe\| + \varepsilon f}$$

(B7)

Taking

$$\beta \geq \frac{|\lambda_{max}(P)|^2}{|\lambda_{min}(B^T P)|^2}$$

(B8)

the last term on the RHS can be made negative by noting $\lambda(X)$ the eigenvalue of matrix X, in which case eqn(B7) simplifies to the first two terms in the RHS of eqn(B7) ie

$$d\mathcal{L}(e)/dt \ \leq \ -<eQe> + \varepsilon M_s(t)g_s(\|e(t)\|, t) f(e, t)$$

(B9)

showing the opposite action of the two terms as discussed in the text. If $f(.)g_s(.)$ is upper bounded over the whole interval, the RHS of eqn(B9) is negative above some threshold value. On the other hand, because $g_s(.)$ is for regular $f(.)$ a higher order term at the origin by construction, the RHS of eqn(B9) is negative close to the origin. So there may exist an interval on the real line where $d\mathcal{L}(e)/dt > 0$ separating two stable domains. If $f(.)g_s(.)$ is growing at infinity, there exists a threshold value above which the system is unstable. So there is an asymptotic stable ball around the origin, and the decay is fixed by the specific functional dependence of the functions $f(.)$ and $g_s(.)$. The next step is to fix the driving term $f(.)$ for determining the upper bound on the time decay of Lyapunov function and finally on the norm of error vector $e(t)$. As $<eQe>$ and $<ePe>$ are equivalent norms, there exists $k>0$ so that $<eQe> \geq k\mathcal{L}(e)$ and eqn(A8) can be replaced by

$$d\mathcal{L}/dt \ \leq \ -k\mathcal{L} + \varepsilon M_s(t) g_s(\mathcal{L}, t) f(\mathcal{L}, t)$$

(B10)

with new dependent variable $\mathcal{L}$, bounded by the solution $Y(t)$ of eqn(B10) with equal sign. Here $f(., t)$ is immediately determined for given functional form of $g_s(., t)$ with prescribed $Y_d(t)$, but a more useful approach is to consider the embedding problem where, for given $M_s(t)g_s(\mathcal{L}, t)$, a correspondence is researched between function spaces $\mathcal{F}$ and $\mathcal{Y}$ to which $f(t)$ and $Y(t)$ respectively belong with $\mathcal{Y}$ fixed by global properties such as continuity and decay for large *t*.

The most general way is to use substitution theorems relating different function spaces[48] and Fixed

Point Theorem[25]. Supposing that Y(t) $\in$ $\mathcal{W}_1^p$ implies that $M_s(t)g_s(\mathcal{L},t) \in \mathcal{W}_1^q$ with Sobolev space $W_m^{n[49]}$, application of Holder inequality shows that there should be $\mathcal{F} \subseteq \mathcal{W}_1^n$ with $n^{-1} = p^{-1} - q^{-1}$, $1 \leq p,q,n < \infty$ which relates the decays of $f(t)$ and of $Y(t)$ for large t. One can then define the driving function $f(t) = \mu Y^\phi$ now implicit in time through the directly measurable error bound and get the equation for the sup bound $Y(t)$ of $\mathcal{L}(e)$

$$Y(t) \leq \exp- \int_0^t k(t')dt'.\Gamma_s^{-1}\left[\Gamma_s(Y_0) + 2\,\mathrm{sgn}(\Gamma_s)\varepsilon/\alpha \int_0^t \exp \right.$$
$$\left. - \int_0^t k(t')dt'.M_s(t')h(t')dt' \right] \quad \text{(B11)}$$

where $\Gamma_s(x) = \int^x dv\, g_s(v)^{(1}$. When $\Gamma_s(.)$ is sub-linear, sgn($\Gamma_s(.)) > 0$, and Y(t) is defined for any [Y(0),t], whereas if $\Gamma_s(.)$ is super-linear, sgn($\Gamma_s(.)) < 0$, and $Y(t)$ is only defined for [Y(0),t] such that the argument of $\Gamma_s(v)^{(1}$ does not change sign, ie $t < t_c$ defined by

$$\Gamma(Y_0) = 2(\varepsilon/\alpha)\int_0^{tc} \exp - \int_0^{t'} k(t')dt'.M_s(t')h(t')dt' \quad \text{(B12)}$$

in order to avoid Lagrange instability in finite time t_c.

More specific results can be found for more explicit constraints. For instance when $g_s(Y,t) \leq b(t)Y^s$ with $b(.) \in \mathcal{L}^\beta$ and $s^{-1} = q^{-1} - \beta^{-1}$ the bounding equation for $Y(t)$ from eqn(B10) becomes a Bernouilli equation with solution

$$y(t) = \exp- kt \left\{ 1 - (\theta-1)\frac{\varepsilon\mu}{k}Y_0^{\theta-1}\int_0^{kt}\exp \right.$$
$$\left. - (\theta-1)u\, M_s(u)b(u)du \right\}^{-1/(\theta-1)} \equiv \exp- kt.U(kt,Y_0,\theta) \quad \text{(B13)}$$

with $\theta = s + \phi$, $y(t) = Y(t)/Y(0)$ and normalized time $u = kt$, which exhibits a non exponential asymptotic decay for $\theta < 1$ and for any Y(0), and a conditional decay when $\theta > 1$ directly depending on the balance between recalling linear spring and repulsive nonlinear force, and defining the attraction domain of eqn(A1) in terms of actual parameters and initial conditions. The result of eqns(B11,B12) shows that asymptotic stability can be found in conjunction with robustness constraint, in contrast to classical approach, and the role of $f(t)$ which drives the behaviour of Y(t) is clearly shown. More importantly, there exists an *equivalence robustness class* for all equations having the same bounding equation as the proposed approach is not focusing on a specific and single eqn(A1) but on a class here defined with few parameters. As indicated above, the present analysis extends in similar form to systems with unknown dynamics by using nonlinear network representation the parameters of which are adaptively constructed to converge toward system representation[35].

The interesting point is the role of the attractive harmonic potential included in the expression of u(.) in eqn(B2) defining the attraction class in S. More generally this suggests the very simple picture of a "test" of eqn(A1) on a prescribed space S by a set of harmonic springs over a base set of S. Evidently if the *smallest* spring is found to be attractive for actual parameters u(.) and d(.) the embedding is realized in **S**. So the embedding is solely controlled by the sign of smallest spring, which is an extremely weak and clearly identified knowledge about the system under study. This opens on the application of *spectral methods* which appear to be particularly powerful here because they are linking an evident physical meaning (the power flow) with a well defined and operating method to construct a fixed point in target space **S**[23]. Obviously the number of dimensions of initial system is irrelevant as long as only the smallest spring force (the smallest eigenvalue of system equation in space S) is required. In this sense such result which generalizes Legendre-Dirichlet theorem is far more efficient than usual Lyapounov method which is a limited algebraic approach to the problem. Another element coming out of previous result is the fact that present approach is particularly well tailored for handling the basic and difficult problem of equivalence, especially asymptotic equivalence where system dynamics reduce for large time to a restricted manifold, and sometimes a finite dimensional one even if initial system dimension is infinite as for instance for turbulent flow in Fluids Dynamics[50]. So asymptotic analysis is also a very powerful tool for studying complex systems, especially when they belong to the class of reducible ones[51]. Such systems are defined by the fact that bifurcation phenomenon which generates the branching toward more complex structure is produced by effects with

characteristic time and space scales extremely different (and much smaller) from base system ones. So the system can be split into large and small components the dynamics of which maintain system global structure from the first when interacting with the second in charge of dissipation because they have lost their phase correlations, hence the name of dissipative structures[17]. As long as they are (usually) small and indistinguishable, initial values of small components are obeying central limit theorem and are distributed according to a Gaussian, see **Figure 4.** However, it is not possible to neglect at this stage their dynamics which can on a (long) time compared to their own time scale act significantly on large components dynamics, and usual Chandrasekhar model[22] does not apply here. Small components dynamics can be asymptotically solved on (long) large components time scale, and injected in large components ones. Then system dynamics are still described by large components, but modified by small components action[52].

APPENDIX C

When the coefficients A_0, B_0 and the bounding function g_s of eqn (B1,B3) are time independent, so that P and Q in Lyapounov equation $PA+A^TP + dP/dt = -Q$ with $A = A_0 - B_0K$, the more general (Lurie type) Lyapunov function[53]

$$V = <x_s, Px_s> +2\eta\tau \int_0^{<D,xs>} \Psi(\xi)d\xi$$

(C1)

can also be used, with D an adjustable vector and $\Psi(.)$ a nonlinear positive sector function with bound τ, ie such that

$$\Psi(\xi)[\Psi(\xi) - \tau\xi] \leq 0 \qquad (C2)$$

with $\tau > 0$ and parameter η to be determined later. Let

$$\Delta u = -\Psi(<D,x_S>)$$

(C3)

Its derivative along nonlinear system trajectories is after substitution given by

$$dV/dt = <x_s, (PA + A^TP)x_s> -2<x_s, PB>\Psi + 2<x_s, PG_s> +2\eta\tau\Psi<D, (Ax_s - B\Psi+G_s)> \qquad (C4)$$

By adding k times eqn(C2) the following bound is obtained

$$dV/dt \leq <x_s, (PA + A^TP)x_s> - 2<x_s, (PB - \eta\tau A^TD - \tau kD)\Psi> - 2(k + \eta\tau <D,B>)\Psi^2 + 2<x_s, PG_s> + 2\eta\tau\Psi<D,G_s>$$

(C5)

Now define the linear transfer function $\Gamma(s) =< D,(sI - \hat{A})^{-1}B >$. If there exists $\eta \geq 0$ so that $-(1/\eta)$ is not an eigenvalue of $\hat{A} = A+(\varepsilon_P/2)I$ and if Popov type condition[54]

$$k^{-1} + \tau\mathscr{R}e[(1 + 2i\eta\omega)\Gamma(i\omega)] \geq \varepsilon_P \eta\tau|\Gamma(i\omega)|^2$$

(C6)

is satisfied for $\omega \in \mathbf{R}_1$, there exists P, L and $\varepsilon_P > 0$ so that the following equalities hold

$$PA + A^TP = -LL^T - \varepsilon_P \eta\tau DD^T - \varepsilon_P P$$

$$PB = \tau(kI + \eta A^T)D - wL$$

(C7)

with $k = 1/2$. Defining $\Gamma(i\omega) = X(i\omega) + iY(i\omega)$, eqn(C7) reduces to circle criterium[23]

$$2(\eta\tau)^{-1} + X\eta^{-1} - 2\omega Y \geq \varepsilon_P(X^2 + Y^2)$$

(C8)

For each $\omega \in \mathbf{R}_1$ the point $(X(i\omega), Y(i\omega))$ should be inside the circle centered at $[1/(2\varepsilon_P \eta), -\omega/\varepsilon_P]$ and passing through the (fixed) points $(X_+, 0)$, $(X_-, 0)$ with

$$X_\pm = (2\varepsilon_P \eta)^{-1} \pm \varepsilon_P^{-1/2}[2(\eta\tau)^{-1} + (4\eta^2\varepsilon_P)^{-1}]^{1/2}$$

(C9)

and with $X_- < 0$, $X_+ > 0$. For ω small $\Gamma \approx -<D,\hat{A}^{-1}B>-i\omega<D,\hat{A}^{-2}B>$ so $Y < 0$ and for $\omega \to \infty$

$$\Gamma \approx i<D,B>/\omega-<D,\hat{A}B/\omega^2$$

(C10)

ie X and Y are decaying to 0. As the circle radius

$$R_c(\omega) = (1/\sqrt{\varepsilon_P})[(\omega^2/\varepsilon_P)+(2/\eta\tau)+(1/4\eta2\_P)]^{1/2}$$

(C11)

increases with ω, it is sufficient for eqn(C9) to be satisfied that both limiting conditions for $\omega \approx 0$ and $\omega \to \infty$, ie $X(0) = -<D,\hat{A}^{-1}B> \leq X+$ and $\eta\tau \leq 1$ be satisfied, which is always possible with convenient choice of parameters.

Using P, L and $\varepsilon_P > 0$ in eqn(C6) and defining w so that $w^2 = 2(k + \eta\tau<D,B>)$, one gets after some manipulations

$$dV/dt \leq -\varepsilon_P[<x_s, Px_s> + \int_0^{<D,xs>}\Psi(\xi)d\xi] - <[Lx_s - w\Psi], [Lx_s - w\Psi]> + 2|x_s|^2(|P|_2 + \eta\tau^2|D|_2^2)|G_s|_2$$

(C12)

Using eqn(16) and defining $\varepsilon(t) = 2(|P|_2 + \eta\tau^2|D|_2^2)M_s(t)$, one finally gets the bounding inequality

$$dW/dt \leq -\varepsilon_P W + [\varepsilon(t)/\lambda_{max}(P)].W^{1/2}g_s(W^{1/2})$$

(C13)

with $W=V/\lambda_{max}(P)$, very clearly showing

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the balance between the stabilizing linear (gain) term and the bound of the nonlinear one. If $g_s(.)$ is upper bounded on the whole interval, the term on the right hand side of eqn(C13) is negative above some threshold value. On the other hand, because $g_s(.)$ is a higher order term at the origin by construction, the right hand side of eqn(C13) is negative close to the origin. So there may exist on the real line an interval where $g_s(.) > 0$, and which separates two domains of stability. If $g_s(.)$ is growing at infinity, there exists a threshold value above which the system is unstable. So there is an asymptotic stable ball around the origin. The decay to 0 is fixed by the specific behavior of $g_s(.)$ and is not necessarily exponential as for the case of linear $g_s(.)$. As a simple example, when $g_s(x) = x^{\lambda s}$ a power law, one gets

$$W(t) \leq W_0 \exp{-\varepsilon_P\, t}\ [1 - W_0^{(\lambda s-1)/2}\ (\varepsilon_P \lambda_{max}(P))^{-1}E(t)\ ]^{2/(\lambda s-1)}$$
$$(C14)$$

with

$$E(t) = \int_0^t \varepsilon(t')\ [1 - \exp{-1/2}(\lambda_s - 1)t']\mathrm{d}t'$$
$$(C15)$$

Asymptotic stability is obtained within the ball $W_0 < (\varepsilon_P \lambda_{max}(P)/\ \hat{E})^{2/(\lambda s-1)}$, where $\hat{E}$ is a normalizing value of $E(t)$. It corresponds to the largest domain for which linear and nonlinear terms become comparable. The singular case $\lambda_s = 1$ corresponding to previous linear power law bound for nonlinear terms leads to strict exponential bound on solution decay in the entire domain $V_0 > 0$. For more general power law bound, absolute asymptotic stability result only extends into a conditional one with a size depending on the competition between the gain in linear control law and the amplitude factor $M_s(t)$, ie the perturbation $d(t)$. Though similar to previous results, in present situation however, robustness is obtained in larger domain. It also shows the advantage of better taylored control law which, as concerns the nonlinear part Δu, can be further improved by adjusting base co-vector D representing the weight given each component of state vector x in more general adaptive setting of neuro-fuzzy nonlinear representation[47].

Using present extension of Popov criterion, the explicit form of driving function $f(\xi) = \xi^{1/2}$ is finally found in eqn(B6) leading to the same bound as in eqn(B11) with $\phi = \frac{1}{2}$, whereas the additional controller takes the simple form $\Delta u = -\Psi(D.e)$ instead of eqn(B6). Analysis of $U = U(kt,Y_0,\theta)$ in eqn(B11) shows that when $(\varepsilon\mu/k)Y_0^{\theta-1} < 1$, $y(t)$ has no singularity over the complete real line, and two cases can occur. If $\varepsilon\mu/k > 1$, Y_0 is always < 1 and larger limit value of Y_0 corresponds to larger exponent θ. So extension of Popov criterion gives a smaller robustness ball than fixed point result. If $\varepsilon\mu/k < 1$, Y_0 can be > 1 in which case conversely, larger limit value of Y_0 corresponds to smaller exponent θ. So extension of Popov criterion is equivalent to fixed point result when taking the smallest allowable exponent value $1/2$. When $(\varepsilon\mu/k)Y_0^{\theta-1} > 1$, there exists a critical time kt_c for which $y(t)$ is singular, showing finite time Lagrange instability. Only finite time boundedness is possible in this case, and given a ball $\mathcal{B}$ it is interesting to determine the largest initial ball $\mathcal{B}(0,Y_0)$ so that its transform $\mathcal{B}(T,Y(T))$ by eqn(A10) satisfies $\mathcal{B}(T,Y(T)) \subseteq \mathcal{B}$ with largest T corresponding to smallest input-output amplification factor. In the same way, smallest exponent corresponds to largest T and largest Y_0 when $\varepsilon\mu/k < 1$, whereas a compromise has to be found when $\varepsilon\mu/k > 1$. So in all cases an adapted controller exists in explicit form from application of Fixed Point Theorem, and is very robust as it only depends on global bounding functions for system equations. These functions may change with the task, so here the system has a consistency link between the task assignment and the nature of its response through the controller guiding system trajectory whatever it starts from toward any trajectory belonging to the task manifold.