Journal of Autonomous Intelligence (2023) Volume 6 Issue 1
doi:10.32629/jai.v6i1.558

ORIGINAL RESEARCH ARTICLE

Financial time series prediction using deep computing approaches

M. Durairaj[1], Ch. Suneetha[2], BH. Krishna Mohan[3*]

[1] *Department of Computer Science and Engineering, Bharathidasan University, Tiruchirappalli 620024, India.*
[2] *Department of Computer Applications, RVR & JC College of Engineering, Guntur 522019, India.*
[3] *Department of Computer Science and Engineering, Bharathidasan University, Tiruchirappalli 620024, India. E-mail: mohanbk28@gmail.com, bkm@rvrjc.ac.in*

ABSTRACT

A financial time series is chaotic and non-stationary in nature, and predicting it outcomes is a very complex and challenging task. In this research, the theory of chaos, Long Short-Term Memory (LSTM), and Polynomial Regression (PR) are used in tandem to create a novel financial time series prediction hybrid, Chaos+LSTM+PR. The first step in this hybrid will determine whether or not a financial time series contains chaos. Following that, the chaos in the time series is modeled using Chaos Theory. The modeled time series is fed into the LSTM to obtain initial predictions. The error series obtained from LSTM predictions is fitted by PR to obtain error predictions. The error predictions and initial predictions from LSTM are combined to obtain final predictions. The effectiveness of this hybrid is examined by three types of financial time series (Chaos+LSTM+PR), including stock market indices (S&P 500, Nifty 50, Shanghai Composite), commodity prices (gold, crude oil, soya beans), and foreign exchange rates (INR/USD, JPY/USD, SGD/USD). The results show that the proposed hybrid outperforms ARIMA (autoregressive integrated moving average), Prophet, CART (Classification and Regression Tree), RF (Random Forest), LSTM, Chaos+CART, Chaos+CART, and Chaos+LSTM. The results are also checked for statistical significance.

Keywords: Deep Learning; Time Series Prediction; LSTM; Chaos; Polynomial Regression; Exchange Rate; Stock Market Index; Commodity Price

ARTICLE INFO

Received: 1 March 2023
Accepted: 9 April 2023
Available online: 4 May 2023

1. Introduction

A financial time series (FTS) consists of regularly recorded observations of financial variables. For example, daily stock market index values, daily commodity prices, and daily exchange rates are FTS. In general, the FTS is chaotic and non-stationary[1]. A time series of chaos is non-linear, deterministic and sensitive to initial conditions[2]. FTS are also noisy, and their statistical properties vary with time. This property makes the prediction impossible[3,4].

Building the right prediction model that can capture the nonlinearity present in the time series is always challenging. So, it reveals that FTS prediction is a challenging and complex task. Several researchers demonstrated that hybrids or ensembles could perform better than stand-alone forecasting models[5–7]. A hybrid forecasting model combines two or more stand-alone forecasting models into an integrated model to improve prediction accuracy and overcome the deficiencies of stand-alone models.

Chaos Theory[8,9] models non-linear FTS by constructing phase space with the help of parameters including lag and embedding dimension. The time difference between two values is represented by a lag. An integrated dimension is the number of variables needed to capture the FTS dynamics.

Applying deep learning approaches can help achieve better prediction accuracy[10,11]. Deep learning, a subset of machine learning, enables Artificial Neural Networks (ANNs) to learn data representation on several abstracted levels (hierarchical learning)[12,13]. The ANNs can build a complex, non-linear function that maps output inputs. These are used to resolve various financial problems such as prediction of the stock market, portfolio optimization, processing of financial information and execution strategies[14]. This is a relatively unexplored field, however.

The LSTM[15] is a Neural Network Recurrent Type (RNN) that will read each time step of the FTS one step at a time. It can learn long-term dependencies to capture the non-linearity present in the time series very well, lead to precise predictions in comparison to linear prediction models, such as automotive regressive integrated moving averages (ARIMA). A new forecasting model based on support vector regression (SVR) with a wrapper-based feature selection approach using multi-objective optimization technique is developed[16].

This paper presents a hybrid model involving Chaos Theory, LSTM, and PR to predict FTS. In this hybrid, first, the FTS is the presence of chaos checked. Later, Chaos Theory can model the chaos present in the FTS. The modeled FTS is input to LSTM to obtain initial predictions. The error series obtained from LSTM predictions is fit by PR to get error predictions. The error predictions and initial predictions from LSTM are added to obtain final predictions from the hybrid model.

The contributions of this research paper include:

1) Chaos+LSTM and Chaos+LSTM+PR are the new chaos-based FTS prediction hybrids.

2) Solutions to three different FTS prediction problems, including commodity price prediction, stock market index prediction, and foreign exchange rate prediction.

3) Comparative study of proposed hybrids with stand-alone time series prediction models with ARIMA, Prophet, Classification and Regression Tree (CART), Random Forest (RF), and LSTM. It also includes a comparative study of the proposed hybrid with other chaos-related hybrids such as Chaos+CART[17] and Chaos+RF[17] found in the literature.

The rest of the paper is described below. Section 2 presents the literature concerned. Later, Section 3 describes the approach proposed in detail. Section 4 describes the experimental design and discusses the results. Section 5 concludes the paper.

2. Review of literature

There are numerous hybrids for financial time series (FTS) found in literature[10,18–23]. The deep learning hybrids for FTS prediction are also found in last two decades of literature and are recently well summarized by Durairaj and Krishna Mohan[24].

2.1 LSTM-based hybrids

This section presents various related LSTM-based hybrids proposed for FTS prediction connected with the previous works. The LSTM-based hybrids are as follows: Bao *et al*.[11] presented a new, 3-stage, wavelet transform (WT), stacked auto-encoders (SAEs) and LSTM deep-learning framework. WT is used in the proposed framework to decompose inventory price series to eliminate noise. Then deep high-level denoising characteristics were generated by the SAEs. Finally, the next day, by using these chosen features, LSTM forecast the closing stock price. The authors concluded that in predictive accuracy as well as in profitability the proposed model exceeded other similar models. Kim and Won[25] proposed hybrid combined LSTM with different GARCH models or more than two econometric model Generalized Auto-Regressive Conditional Heteroskedasticity (GARCH). The authors discovered that the proposed hybrid outperformed other bench-mark models. Baek and Kim[26] proposed the ModAugNet framework to forecast FTS. In this model, LSTM is used for two purposes, including the prevention of overfitting and prediction. The results confirmed the ModAugNet framework could return good forecasting accuracy. Cao *et al*.[27] proposed hybrid forecasting models, namely EMD-LSTM and CEEDMAN-LSTM. Both Empirical Mode Decomposition (EMD) and Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEDMAN) are used to reduce the impact of noise in FTS by decomposing the original series. After then the denoised series are input to LSTM to obtain predictions. The authors concluded that the hybrids proposed could provide a precise one-step forecast. Zhang *et al.*[14] suggested a new generative adversary network architecture (GAN). Multi-layer Perceptron (MLP) is the discriminator in this architecture and the LSTM generates a prediction for closing stock prices. The authors concluded that the model proposed outperformed stand-alone learning and deep learning.

2.2 Chaos-based hybrids

Table 1 presents the chaos-based hybrids proposed for FTS prediction found in literature. All these works concluded that the proposed chaos-based hybrid was distinct.

Table 1. Chaos-based hybrids in the literature for prediction of financial time series

Year	Author(s)	Chaos-based hybrids
2003	Pavlidis *et al.*[28]	Chaos Theory Hybrid Methodology, ANN, and Clustering PSO/DE
2010	Huang *et al.*[29]	SVR+Chaos
2014	Pradeepkumar and Ravi[29]	ANN+PSO*+Chaos, PSO+ANN*+Chaos, PSO+ANN*+Chaos
2016	Pradeepkumar and Ravi[17]	QRRF*+Chaos, QR+Chaos, RF+Chaos, CART+Chaos, CART-EB+Chaos
2017	Pradeepkumar and Ravi[31]	TreeNet+Chaos, LASSO+Chaos, RFTE+Chaos, MARS*+Chaos
2017	Ravi *et al.*[32]	MLP+MOPSO+Chaos, MLP+NSGA-II*+Chaos

* = Winner Hybrid

ANN: artificial neural network; QR: quantile regression; QRRF: quantile regression random forest; RF: random forest; CART: classification and regression tree; CART-EB: CART ensemble; RFTE: RF tree ensemble; PSO: particle swarm optimization; DE: differential evolution; MOPSO: multi-objective PSO; MARS: multivariate adaptive regression splines; Lasso: least absolute shrinkage selection operator; NSGA-II: non-dominated sorting genetic algorithm-II.

The literature states that hybrids based on LSTM and chaos can yield exact predictions compared to stand-alone models. As the FTS is chaotic and deep learning models can capture non-linearity present in the FTS, this paper presents chaos-based LSTM hybrids to predict FTS.

3. Proposed approach

In the proposed hybrid, Chaos+LSTM+PR, a financial time series is checked for the presence of chaos. Lyapunov exponent[32] is used for this purpose. Chaos Theory is subsequently used to build space from scalar time series. Optimal lag and optimal embedding size values are required for construction of phase space[33,34]. Akaike Information Criterion (AIC)[35] is used for selecting optimal lag from time series. Cao's method[36] is used for obtaining optimal embedding dimension. Once optimal lag and optimal embedding dimension are obtained from time series, phase space can be reconstructed using Chaos Theory. Later, LSTM is used for obtaining initial predictions, and finally, Polynomial Regression is used to finetune predictions. The proposed hybrid is compared with ARIMA[37], Prophet (https://facebook.github.io/prophet/), CART, RF, LSTM, Chaos+CART, Chaos+RF and Chaos+LSTM.

Table 2. Notations used in proposed approach

Notation	Interpretation
l	Optimal lag
m	Optimal embedding dimension
y_t	Actual observation at time t
et	Time at error obtained t
$\dot{e}t$	Prediction of error at a time t
$\dot{y}t$	First forecast on time t
$\ddot{y}t$	Time at final prediction t
$f_1(.)$	Non-linear function used by LSTM to obtain predictions
$f_2(.)$	Linear function used by PR to obtain predictions

The process describes the proposed hybrid approach.

- Let Y = $\{y_1, y_2, y_3, ..., y_k, y_{k+1}, ..., y_N\}$ be a series of times N observations recorded at times t = {1, 2, 3, ..., k, k + 1, ..., N}. Then perform the following:
- Check Y if chaos is present. If chaos is present, obtain optimal lag (l) and optimum dimension of embedding (m) from Y.
- Once optimal lag and embedding dimension values are obtained, reconstruct phase space from Y.
- After phase space is reconstructed, partition Y into $Y_{Train} = \{y_t; t = lm + 1, lm + 2, ..., k\}$ and

$Y_{Test} = \{yt; t = k + 1, k + 2, ..., N\}$.

- Input Y_{Train} to LSTM, train LSTM and get predictions of the initial training set Eq 1.

$$Y^{\cdot}_t = f_1(y_{t-1}, y_{t-2l}, ..., y_{t-ml}) \tag{1}$$

where $t = lm + 1, lm + 2, ..., k$.

Obtain initial test set predictions by input YTest to trained LSTM by replacing $t = \{k + 1, k + 2, ..., N\}$ in Eq 1.

Compute training set of prediction errors using Eq 2 and test set of prediction errors by replacing $t = \{k + 1, k + 2, ..., N\}$ in Eq 2.

$$e_t = y_t - y^{\cdot}_t, \tag{2}$$

where $t = lm + 1, lm + 2, ..., k$.

- Fit Polynomial Regression to training set of errors and obtain training set error predictions using Eq 3. Similarly, fit PR to test set of errors and obtain test set error predictions by replacing $t = \{k + 1, k + 2, ..., N\}$ in Eq 3.

$$e^{\cdot}_t = f_2(e_t) \tag{3}$$

where $t = lm + 1, lm + 2, ..., k$.

- Add training set initial predictions and training set error predictions to obtain final training set predictions using Eq 4. Similarly, add test set initial predictions and test set error predictions to obtain final test set predictions by replacing $t = \{k + 1, k + 2, ..., N\}$ in Eq 4.

$$y^{\cdot\cdot}_t = y^{\cdot}_t + e^{\cdot}_t \tag{4}$$

where $t = lm + 1, lm + 2, ..., k$.

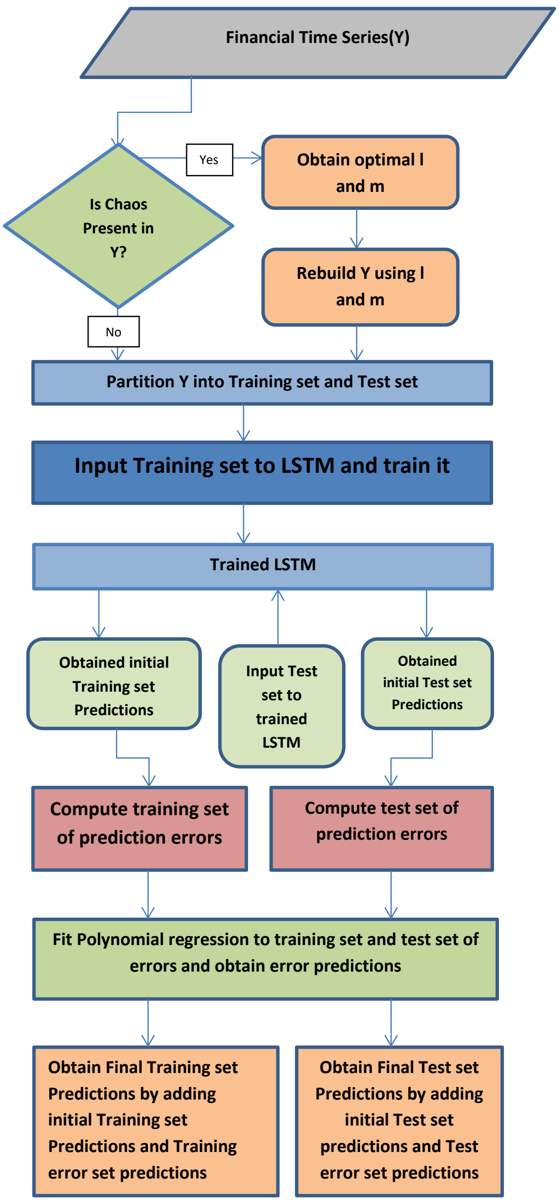

Figure 1. Proposed approach.

4. Used data sets

Different data sets are used in this paper to observe the effectiveness of proposed hybrids. These

Table 3. Datasets used

Data set	Dates	Count	Training set	Test set
Crude Oil Price (USD)	02-Jan-1990 to 29-Jan-2021	7,890	6,312	1,578
Gold Price (USD)	02-Jan-1990 to 29-Jan-2021	7,907	6,326	1,581
Soybeans Price (USD)	02-Jan-1990 to 31-Jan-2021	8,063	6,451	1,612
Nifty 50 Stock Price	06-Nov-1995 to 29-Jan-2021	6,281	5,025	1,256
Shanghai Composite Index	20-Dec-1990 to 29-Jan-2021	7,362	5,890	1,472
S&P 500 Stock Index	02-Jan-1990 to 29-Jan-2021	7,831	6,265	1,566
INR/USD	02-Jan-1990 to 29-Jan-2021	8,093	6,475	1,618
JPY/USD	02-Jan-1990 to 29-Jan-2021	8,101	6,481	1,620
SGD/USD	02-Jan-1990 to 29-Jan-2021	8,101	6,481	1,620

Table 4. Statistical descriptive measures

Data	Min	Mean	Median	Max	SD	Data	Skewness	Kurtosis
1. Crude Oil Price (USD)								
All data	7,890	−37.63	47.70264	41.405	145.29	28.83285	0.73005	−0.49341
Training set	6,312	10.72	46.91822	30.35	145.29	31.72228	0.75695	−0.81044
Test set	1,578	−37.63	50.84032	50.915	76.41	10.91546	−0.63922	2.93409
2. Gold Price (USD)								
All data	7,907	253	797.95469	465	2,069.4	515.3496	0.53530	−1.27063
Training set	6,326	253	648.62458	388.1	1,888.7	457.3733	1.17048	−0.13432
Test set	1,581	1,070.8	1,395.46405	1,324.2	2,069.4	212.11469	1.33137	0.90218
3. Soybeans Price (USD)								
All data	8,063	410	833.65716	775	1,764.75	301.45914	0.764002	−0.32651
Training set	6,451	410	803.83398	658.25	1,764.75	327.03820	0.977824	−0.32905
Test set	1,612	803.5	953.00537	936.75	1,430	93.48183	1.718141	4.81299
4. Nifty 50 Stock Price								
All data	6,281	788.15	4,719.14243	4,332.95	14,730.95	3,542.53831	0.62894	−0.77501
Training set	5,025	788.15	3,330.31720	2,598.05	8,996.25	2,335.60170	0.60453	−0.87574
Test set	1,256	6,970.6	10,275.54908	10,495.65	14,730.95	1,530.64147	0.01037	−0.27984
5. Shanghai Composite Index								
All data	7,362	104.39	1,994.61469	1,924.3	6,092.06	1,075.78886	0.50195	0.08418
Training set	5,890	104.39	1,702.09134	1,526.139	6,092.06	989.68912	1.13437	2.17665
Test set	1,472	2,464.36	3,165.10552	3,114.73	5,166.35	395.55923	1.82079	5.53021
6. S&P 500 Stock Index								
All data	7,831	295.450012	1,335.66791	1,210.930054	3,862.959961	757.17977	0.93029	0.42257
Training set	6,265	295.450012	1,023.83847	1,110.469971	2,032.359985	418.18596	−0.09975	−0.8005
Test set	1,566	1,833.40002	2,583.18476	2,577.915039	3,862.959961	471.20240	0.49377	−0.57165
7. INR/USD								
All data	8,093	16.8	46.88723	45.5	76.975	14.00687	0.14052	−0.49158
Training set	6,475	16.8	41.56766	43.73	68.805	10.00020	−0.42131	0.207695
Test set	1,618	61.3580	68.17536	67.41149	76.975	3.83363	0.35588	−0.88452
8. JPY/USD								
All data	8,101	75.82	110.50040	109.98	159.88	15.10520	0.03674	0.43326
Training set	6,481	75.82	110.26933	109.54	159.88	16.648432	0.06609	−0.09937
Test set	1,620	99.89	111.42484	110.62	125.62	5.5728100	0.54998	−0.33050
9. SGD/USD								
All data	8,101	1.2006	1.51456	1.4703	1.9085	0.18029	0.16609	−1.246123
Training set	6,481	1.2006	1.55079	1.5905	1.9085	0.18393	−0.26470	−1.123532
Test set	1,620	1.2976	1.36961	1.3644	1.4598	0.03072	0.327661	−0.215345

daily datasets of 30 years approximately include:

Three commodity prices in US dollars namely Crude Oil Price, Gold Price, and Soyabean Price are collected from Investing.com.

Three securities equities, namely S&P 500, Nifty 50, and Shanghai Composite Index, are collected from Investing.com.

Three foreign exchange rates, namely INR/USD, JPY/USD, SGD/USD are collected from Federal Reserve.

Table 3 presents these datasets along with corresponding dates, number of observations, training set, and test set. Here, the problem of predicting financial time series is modeled as a supervised problem of learning. So, each data set is divided into a training set (80% of observations) and a test set (20% of observations). All of these datasets are checked for chaos, and it is found that chaos is present in each dataset. Later, phase space is reconstructed from each dataset using a related optimal lag and optimal embedding dimension.

Table 4 presents various statistical descriptive measures such as minimum, mean, median, maximum, default, skewedness and kurtosis for the datasets. Data asymmetry measures skewness. The value Zero indicates the data is perfectly symmetric. The positive number shows that on the side above the average of the distribution the tail is extended. The negative figure shows that the tail is longer below the mean on the side of the distribution. It is clear from the table that the tails of all datasets distribution are spread on the above-average side. When contrasted to the normal distribution, the kurtosis describes how peaked or flat a distribution is. A positive kurtosis suggests a peaked distribution, whereas a negative kurtosis indicates a flat distribution. The datasets except for Shanghai Composite Index, S&P 500 Stock Index, and JPY/USD have relatively flat distribution.

5. Tasks performed and tools employed

During experimentation, various tasks are performed. **Table 5** presents such tasks along with tools employed. It should be noted that the best p, d, and q values of the ARIMA model are obtained using auto ARIMA(.) from "pmdarima" module of python. And also, the estimate EmbeddingDim(.) method from "nonlinear time-series" package implemented Cao's method[36].

6. Performance measures used

The suggested hybrid's performance is measured using four performance measures: mean

Table 5. Tasks performed and tools employed

Task	Package/module	Function/measure/class	Tool used
Checking for the presence of chaos	nolds	lyap_r(.)	Python
Finding optimal lag	–	AIC	Gretl
Finding optimal embedding dimension	nonlinearTseries	estimateEmbeddingDim(.)	R
Importing data	pandas	read_csv(.)	Python
Partitioning data	scikit-learn	train_test_split(.)	Python
Fitting ARIMA to data	statsmodels	ARIMA(.).fit(.),forecast(.)	Python
Fitting Prophet to data	fbprophet	Prophet(.).fit(.),predict(.)	Python
Fitting LSTM to data	keras	LSTM(.),predict(.),	Python
Fitting PR to data	scikit-learn	PolynomialFeatures(.) LinearRegression(.),predict(.)	Python
Computing MSE	scikit-learn	mean_squared_error(.)	Python
Computing Dstat	–	–	Python
Computing Theil's U	–	–	Python
Checking for statistical significance	forecast	dm. test(.)	R

squared error (MSE), mean absolute percentage error (MAPE), directional change statistic (Dstat), and Theil's inequality coefficient (Theil's U).

6.1 Mean squared error (MSE)/mean absolute percentage error (MAPE)

By measuring the average of squared errors, the MSE (see Eq 5) determines the prediction of the reaction by the model[38]. The MAPE[38] calculates the absolute numbers of errors in percentage terms to determine how well the model predicts the response. An MSE/MAPE score near 0 suggests that the suggested model could produce predictions that are more accurate than the observed data.

$$MSE = \frac{\sum_{t=1}^{N}(y_t - y'_t)^2}{N} \tag{5}$$

$$MAPE = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{y_t - y'_t}{y_t}\right| \tag{6}$$

where N is the series of number as well as the predicted values are noted at time t. In the result section, Tables shows the comparative results.

6.2 Change in statistical directionals

Yao *et al.*[1] developed a measure (expressed in percentages) namely Dstat (see Eq 7) to measure the directional change of time series. Higher the value of Dstat measurements can be expressed in terms of,

$$Dstat = \frac{1}{N}\sum_{t=1}^{N} a_1 * 100\%$$
$$a_t = \begin{cases} 1, if(y_{t+1} - y_t) * (\widehat{y_{t+1}} - y_t) \geq 0 \\ 0, otherwise \end{cases} \tag{7}$$

Better time-series developments are discovered by the model with a higher Dstat score. The normalized mean square blunders only establish prediction uniquely as far as levels with the aim that these series are attractive. As a result, a person's ability to foretell the future it is possible to be judged by the precision of their angle (Dstat) predictions and the advancements in sign.

6.3 Theil's inequality coefficient

The inequality coefficient of Theil alluded to be U, decides how much nearer an estimate time series is to the ongoing time series. The U worth by and large ranges from 0 to 1. A worth of zero is joined with an ideal expectation, e.g., implying that $U = 0$ fits well with all perceptions. Also, a worth of one is related with a figure that on normal has a similar blunder as a "gullible" no change gauge, i.e., $U = 1$ demonstrates the outcome is low.

$$U = \frac{\sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2}}{\sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t)^2 + \frac{1}{N}\sum_{t=1}^{N}(\hat{y}_t)^2}} \tag{8}$$

7. Experimentation and results discussion

While experimenting with the datasets, various parameters are obtained, and some parameters are utilized in common. **Table 6** presents the optimal values for chaotic parameters obtained. λ is used to determine the presence of chaos in a dataset. $\lambda \geq 0$ denotes the presence of chaos. From the table, it is clear that all of the datasets have chaos. The optimal chaotic parameters such as lag (l) and dimension of embedding (m) are also presented in **Table 6.** The optimal parameters for ARIMA (p, d, q) will be presented in respective sections. The commonly used parameters for all datasets are as follows. The LSTM architecture used here consists of one fully connected dense layer of 50 nodes. Each node is with activation function of ReLU. For the LSTM to be trained for 500 epochs, adam optimizer is used with MSE as a loss function. Scaled values using MinMaxScaler are input to LSTM, Chaos+LSTM and Chaos+LSTM+PR. While modeling errors using PR, second-degree polynomial regression is used.

The results are described in the following terms. It is important to note that, for each dataset, the proposed hybrid (Chaos+LSTM+PR) is compared with ARIMA, Prophet, LSTM, CART, RF, Chaos+CART[17], Chaos+RF[17] and Chaos+LSTM in terms of MSE, MAPE, Dstat and Theil's U.

Table 6. Chaotic parameters

Dataset	λ	l	m
Crude Oil Price	0.001618635	4	9
Gold Price	0.000222457	10	8
Soyabeans Price	0.003601366	10	8
Nifty 50	0.002267289	10	8
Shanghai Composite Index	0.003585269	8	7
S&P 500	0.001685243	1	9
INR/USD	0.00099022	6	10
JPY/USD	0.003709193	1	8
SGD/USD	0.002180623	2	8

7.1 Crude oil

Table 7 shows the results of the Crude Oil Price in the US dollars test set. In terms of MSE, MAPE, Dstat, and Theil's U, the proposed hybrid (Chaos+LSTM+PR) clearly outperforms ARIMA (2,1,0), Prophet, LSTM, CART, RF, Chaos+CART, Chaos+RF, and Chaos+LSTM in the table. It shows the predictions are closer to actual values. This is also depicted by **Figure 2**. The figure also depicts the predicted values of LSTM and Chaos+LSTM. However, Chaos+LSTM could predict well compared to Chaos+CART, Chaos+RF,CART, RF, and LSTM in terms of MSE and MAPE.

Table 7. Results of the crude oil test set

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	122.17013	15.629139	52.758402	0.0226266
Prophet	2079.5150	43.669223	50.158528	0.1817419
CART	5.9811462	3.4417719	48.680405	0.0011074
RF	5.9069172	3.4390847	48.826886	0.0007223
LSTM	5.7178772	3.30012595	49.587824	0.000727
CNN	4.923178	3.212694251	50.22194039	0.000761211
Chaos+CART	5.8593575	3.4006169	51.616994	0.0010846
Chaos+RF	5.02599745	5.489568998	50.34876347	0.001882526
Chaos+LSTM	4.433305	2.5539713	55.694990	0.0009945
Chaos+LSTM+PR	2.55E-07	1.0007893	80	2.87E-11

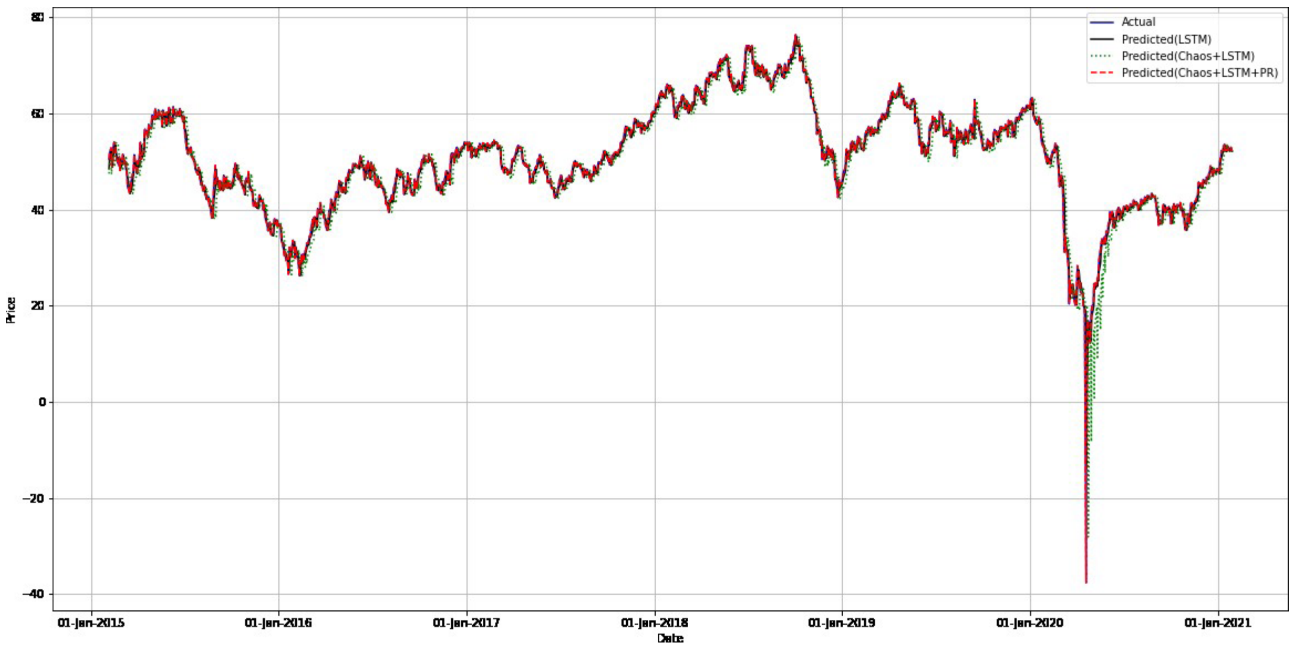

Figure 2. Crude oil test predictions using the suggested hybrid.

Table 8. Results of the Gold Price test set

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	122.17013	15.629139	52.758402	0.0226266
Prophet	2079.5150	43.669223	50.158528	0.1817419
CART	5.9811462	3.4417719	48.680405	0.0011074
RF	5.9069172	3.4390847	48.826886	0.0007223
LSTM	5.7178772	3.30012595	49.587824	0.000727
CNN	4.923178	3.212694251	50.22194039	0.000761211
Chaos+CART	5.8593575	3.4006169	51.616994	0.0010846
Chaos+RF	5.02599745	5.489568998	50.34876347	0.001882526
Chaos+LSTM	4.433305	2.5539713	55.694990	0.0009945
Chaos+LSTM+PR	0.0031655	0.0040988	87.456	0.009462e-10

Figure 3. Test set predictions of Gold Prices using the proposed hybrid.

7.2 Gold Price (USD)

Test set results are shown in **Table 8** for the Gold Price in US dollars. Chaos+LSTM+PR outperforms ARIMA (2,1,1), Prophet, LSTM and CART in the table in terms of MSE and MAPE, Dstat and Theil's U in terms of MSE, Dstat and Theil's U. It demonstrates that the forecasts are more accurate. **Figure 3** also shows this. The anticipated values of LSTM as well as Chaos+LSTM are also shown in the graphic. In terms of MSE and MAPE, Chaos+LSTM did well as Chaos+CART, Chaos+RF, CART, RF, as well as LSTM.

7.3 Soyabean Price

In the Soyabean Price US dollar test set, the findings are shown in **Table 9**. Chaos+LSTM+PR is clearly superior to ARIMA (0,1,0), Prophet, LSTM and CART in terms of MSE and MAPE, as well as in terms of Theil's U, compared to ARIMA (0,1,0), CART, and Chaos+LSTM. It demonstrates that the forecasts are more accurate. **Figure 4** depicts this as well. In terms of MSE and MAPE, Chaos+LSTM did not perform as well as Chaos+CART, Chaos+RF, CART, RF, or LSTM.

Table 9. Results of test set of Soyabean Prices

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	36,277.433250	15.161464	50.775915	0.016791
Prophet	317,374.435389	36.564604	50.900062	0.099628
CART	492.448338	1.732297	55.307262	0.000268
RF	186.908517	1.072228	51.707014	0.000101
LSTM	124.167554	0.817279	51.024208	0.779597e-05
CNN	110.1634494	0.69731048	53.0726257	0.89e-05
Chaos+CART	105.229258	1.730958	55.493482	0.0002632
Chaos+RF	99.237605	1.065448	54.203600	0.0001015
Chaos+LSTM	97.197407	0.2306449	58.893234	0.000890
Chaos+LSTM+PR	2.43608e-05	0.000306	93.785	0.0028e-12

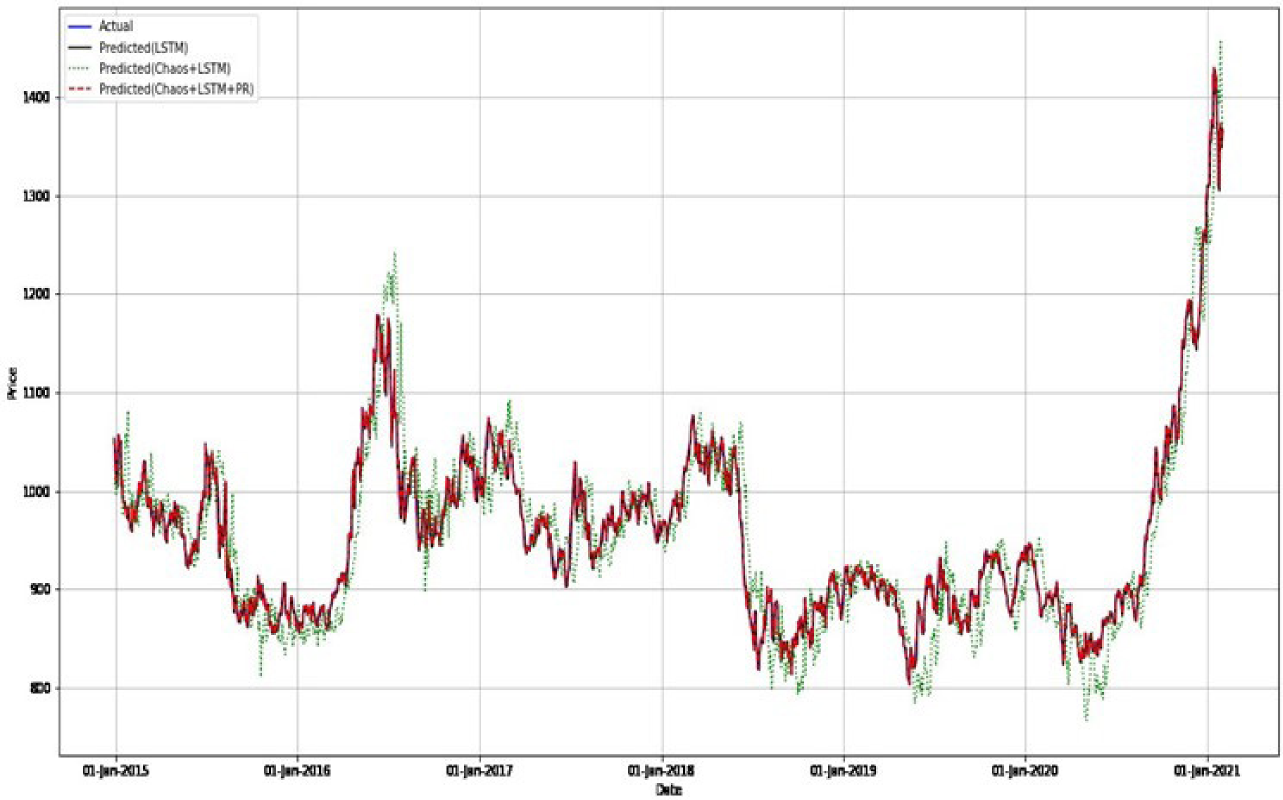

Figure 4. Predictions of test set of US soybeans using the proposed hybrid.

7.4 Shanghai Composite Index

Results are shown in **Table 10**. The proposed hybrid (Chaos+LSTM+ PR) surpasses ARIMA (3,1,3), Prophet, LSTM, CART, RF, Chaos+CART, Chaos+RF, as well as Chaos+LSTM in the table in terms of MSE, MAPE, Dstat, and Theil's U. It demonstrates that the forecasts are more accurate. **Figure 5** depicts this as well. In terms of MSE and MAPE, Chaos+LSTM did perform well as Chaos+-CART, Chaos+RF, CART, RF, or LSTM.

Table 10. Results of test set of Shanghai Composite Index

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	668,777.360368	19.1875061	53.840924	0.0272434
Prophet	2,265,404.326675	87.2517349	51.597552	0.1726264
CART	8,076.550985	1.950799	53.8409245	0.000397
RF	3,623.417136	1.256007	49.4901427	0.000178
LSTM	2,956.134975	1.1817620	55.130523	0.000146
CNN	1,915.863751	1.147577864	57.6546567	0.000190871
Chaos+CART	1,433.170556	1.0793280	59.2685248	0.000365
Chaos+RF	1,359.312050	1.0246783	69.5581237	0.000176
Chaos+LSTM	1,254.767975	1.007058429	73.6940856	0.001777
Chaos+LSTM+PR	1.76715	1.00342	88.567	0.000129e-09

Figure 5. Predictions of test set of Shanghai Composite Index using the proposed hybrid.

7.5 Nifty 50 Stock Index

The findings are shown in **Table 11**. Chaos+LSTM+PR surpasses ARIMA (0,1,1), Prophet, LSTM and CART as well as the proposed hybrid (Chaos+LSTM+PR) in the table in terms of MSE, MAPE, Dstat and Theil's U. It demonstrates that the forecasts are more accurate. **Figure 6** also depicts this. According to MSE and MAPE, Chaos+LSTM did perform well as Chaos+CART, Chaos+RF RF, CART, or LSTM.

Table 11. Nifty 50 Stock Index test results

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	3,400,655.703089	17.260416	55.139442	0.018258
Prophet	849,534.677585	6.739681	52.270916	0.003937
CART	451,114.427545	19.200982	87.808764	0.024733
RF	418,860.943174	18.087281	87.808764	0.022805
LSTM	16,129.754511	10.850068	88.836653	0.499417e-05
CNN	6,957.30123	9.946663737	89.03187251	0.000328416
Chaos+CART	4,510.369836	8.194321	90.569721	0.0247330
Chaos+RF	4,232.770306	8.0233875	92.888446	0.0230679
Chaos+LSTM	1,560.525616	7.837223	95.677290	0.008003
Chaos+LSTM+PR	43.639324	1.2165	94.456	0.002894e-08

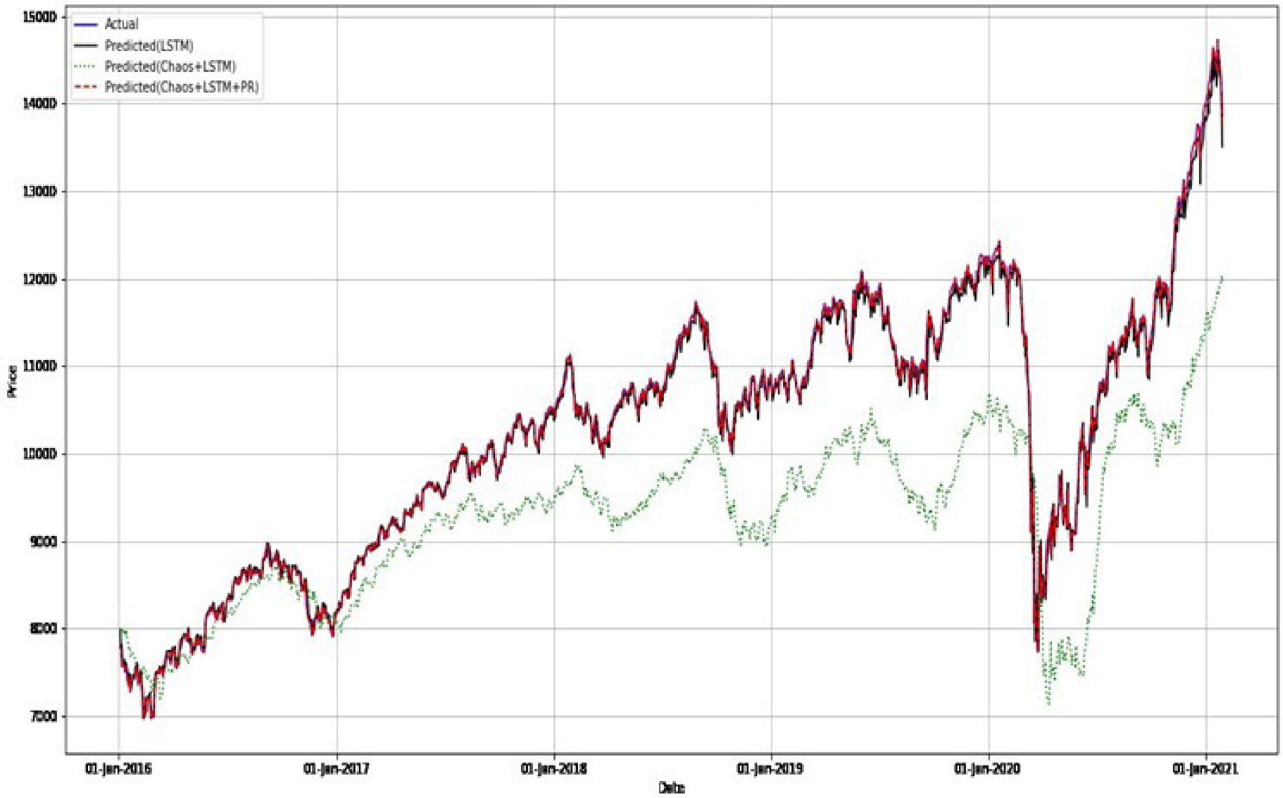

Figure 6. Predictions of test set of Nifty 50 Stock Index.

7.6 S&P 500 Stock Index

The test results for the S&P 500 Stock Index are shown in **Table 12**. MCompared to Chaos+-CART and Chaos+RF, the proposed hybrids Chaos+LSTM and Chaos+LSTM+PR can outperform in terms of MSE, MAPE, Dstat, and Theil's U. CNN outperforms all other ARIMA, Prophet, CART, RF and LSTM models when it comes to MSE, MAPE, and Theil's U. It demonstrates that the forecasts are more accurate than the actual numbers. **Figure 7** depicts this as well. It's worth noting that the Prophet performed the worst of all the techniques presented in the table.

Table 12. S&P 500 Stock Index test results using the proposed two-stage hybrids

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	246,490.773162	15.918238	57.444089	0.020656
Prophet	645,978.130974	11.909248	54.376996	0.011875
CART	524,902.738023	27.451097	96.549520	0.047702
RF	531,831.296643	27.811611	95.846645	0.048434
LSTM	841.164380	10.803366	97.437699	0.114755e-05
CNN	353.6204	9.837133625	98.47923323	0.002729612
Chaos+CART	252.627397	7.453308	98.613418	0.047702
Chaos+RF	151.715480	6.905680	98.846645	0.048624
Chaos+LSTM	70.470136	5.690652	99.309904	5.137796e-05
Chaos+LSTM+PR	1.47217	1.0049732	92.5632	0.0051e-09

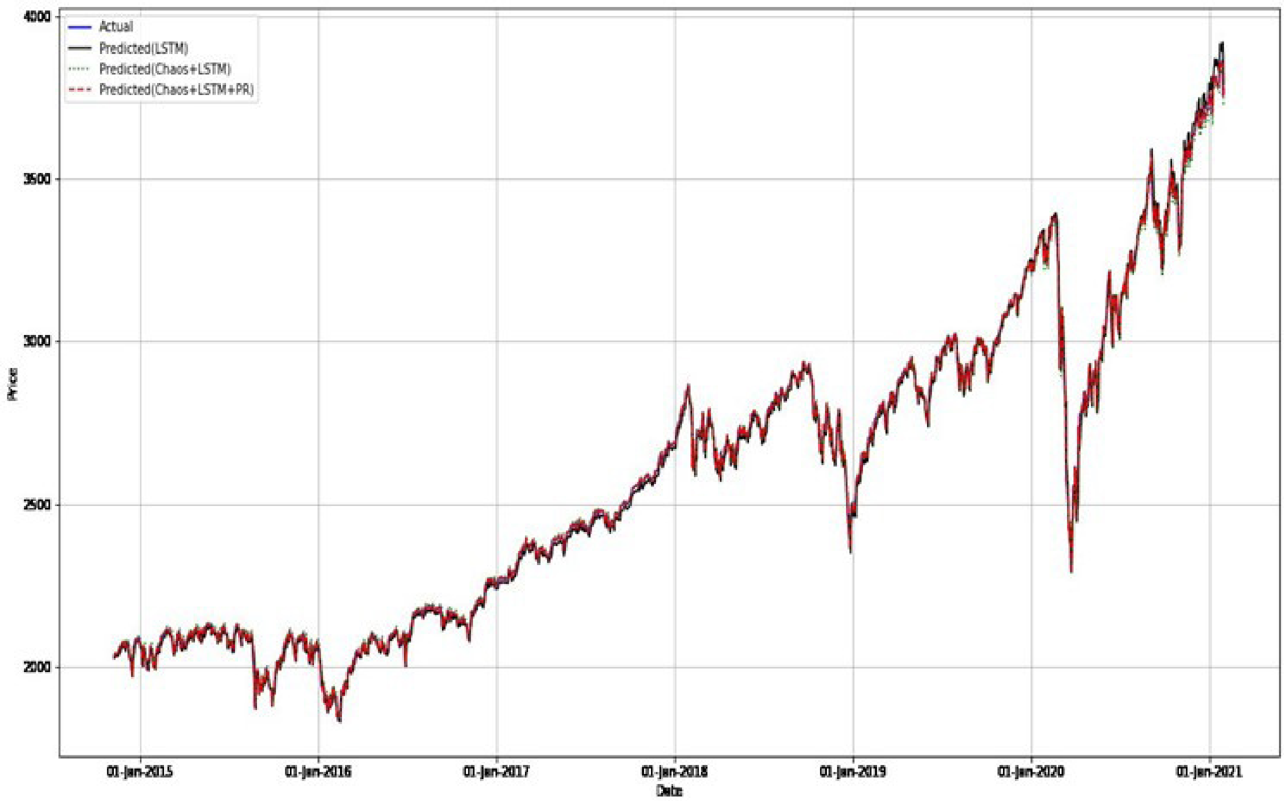

Figure 7. A two-stage hybrid model for the S&P 500 test set.

7.7 INR/USD

Table 13 displays the test set's INR/USD results. Chao+LSTM+PR outperforms ARIMA (3,1,2), Prophet, LSTM, RF, Chaos+CART, Chaos+RF, and Chaos+LSTM in the table in terms of MSE, MAPE, Dstat, and Theil's U. It serves as evidence that the projections are more accurate than previously believed to be the case.

Table 13. The results of the INR/USD test set

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	24.682253	5.644728	50.587507	1.000507
Prophet	19.351410	4.903262	52.690166	0.001971
CART	18.622667	4.976640	91.774891	0.002088
RF	18.081244	4.503860	74.582560	0.002023
LSTM	10.059231	2.260106	94.969078	0.353172
CNN	9.406507115	1.415129313	95.64007421	0.000153046
Chaos+CART	8.243653	1.208711	97.661719	0.002264
Chaos+RF	8.119208	1.499516	98.902288	0.002028
Chaos+LSTM	6.625058	0.893032	99.834879	0.718151
Chaos+LSTM+PR	2.4521e-08	1.039854	95.8744	0.005e-12

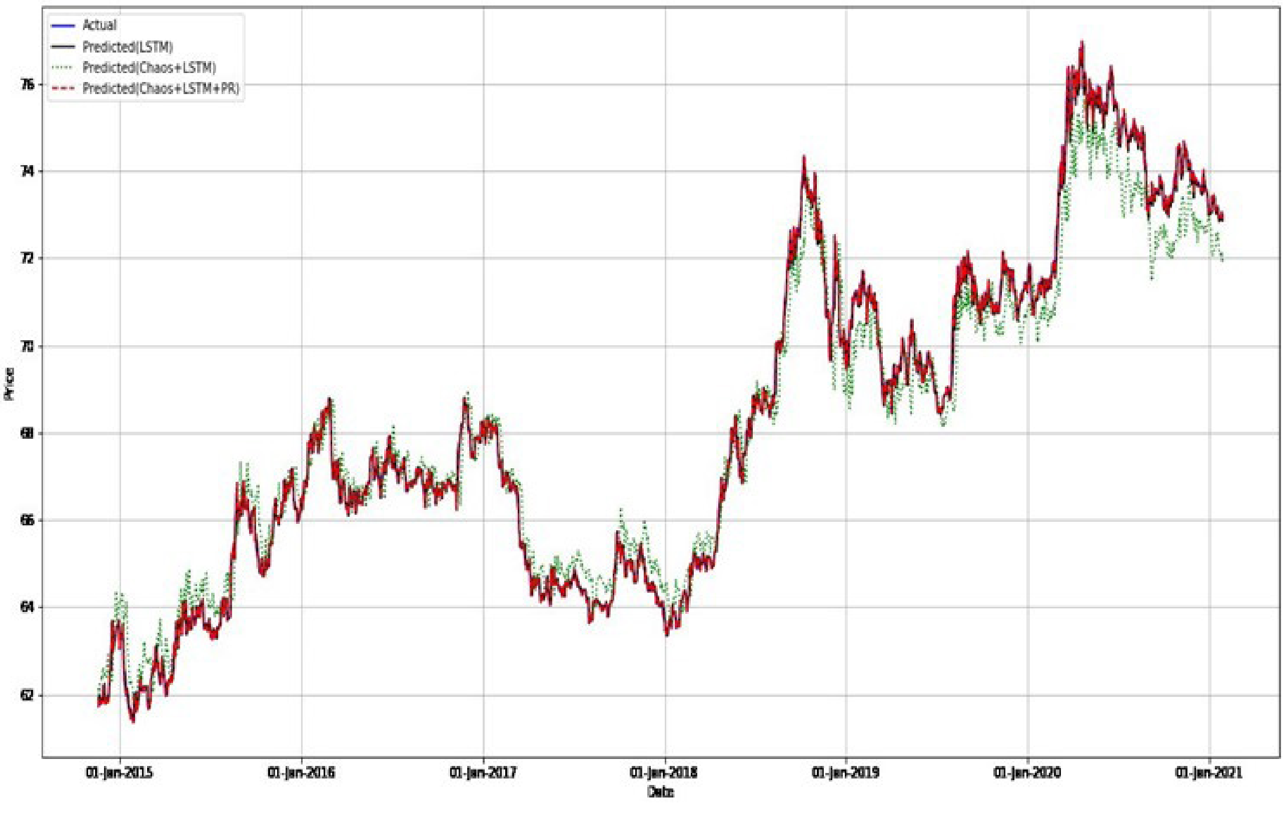

Figure 8. Prediction of INR/USD test set using the hybrid model.

7.8 JPY/USD

The findings of the JPY/USD test set are shown in **Table 14**. A couple of proposed hybrid algorithms, Chaos+LSTM performed in terms of MSE and MAPE and Chaos+LSTM+PR, perform better than in terms of Dstat, as well as Theil's U, and might outperform all other ARIMA, Prophet, CART, RF, and CNN models. It's worth noting that the Prophet performed the worst of all the techniques presented in the table.

The predictions of both proposed methodologies versus actual test set values are shown in **Figure 9**. Both approaches anticipate significantly different values from the actual test set values, as seen in the graph.

Table 14. The results of the JPY/USD test set

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	20.522047	3.029495	50.833848	1.000817
Prophet	123.833278	8.199826	49.289684	1.005242
CART	0.903238	0.635285	51.760345	1.629257e-05
RF	0.464442	0.441553	49.845583	1.865966e-05
LSTM	0.859494	0.174147	68.054354	0.0443902e-05
CNN	0.34434871	0.194904509	69.66028413	0.41e-05
Chaos+CART	0.892972	0.133950	75.895614	0.587788e-05
Chaos+RF	0.464889	0.1445031	78.857319	0.867801e-05
Chaos+LSTM	0.361535	0.0375804	88.424953	0.451573e-05
Chaos+LSTM+PR	1.312878e -08	1.003427	98.5667	0.274069e-13

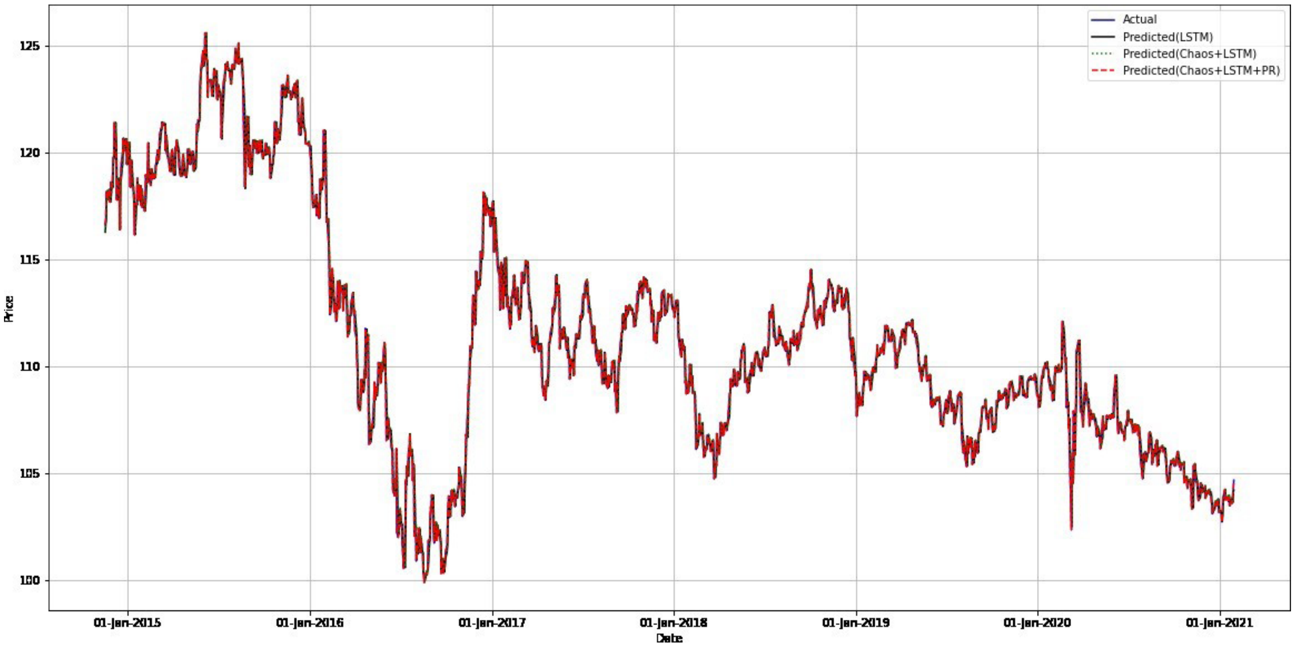

Figure 9. Predictions of test set of JPY/USD using the proposed hybrid.

7.9 SGD/USD

The results of the SGD/USD test set are shown in **Table 15**. Chaos+ LSTM and Chaos+LSTM+PR is clearly superior to ARIMA (0,1,0), Prophet, LSTM, CNN, RF and CART as well as Chaos+RF and Chaos+LSTM in the table of performance metrics. It reveals that the forecasts are closer to the actual values than previously thought. It shows the predictions are strongly correlated in comparison to the actual values shown in **Figure 10**.

8. Diebold and Mariano test

Finally, a formal evaluation is added, that is Diebold and Mariano, which tests if Chaos+LSTM+PR performs substantially differently on average from other forecasting models. The absolute values of the Diebold-Mariano test statistics are shown in **Table 16** on nine datasets. The table clearly shows that Chaos+LSTM+PR outperforms all other models on all datasets.

Table 15. The results of the SGD/USD test set

Forecasting model	MSE	MAPE	Dstat	Theil's U
ARIMA	20.024654	12.260339	51.822112	0.007313
Prophet	10.049610	19.039075	51.142680	0.015482
CART	4.499225e-05	10.384234	55.095738	0.198772e-05
RF	2.754348e-05	9.291158	48.857319	0.339461e-06
LSTM	2.378308e-05	4.270992	69.833848	0.345399e-06
CNN	2.66E-05	6.28453484	58.78381717	0.08E-06
Chaos+CART	4.544950e-05	0.381212	76.948733	0.210897e-05
Chaos+RF	2.695724e-05	0.287179	79.351451	0.183174e-06
Chaos+LSTM	1.842131e-05	0.339434	85.216182	0.023645e-05
Chaos+LSTM+PR	1.175296e-12	0.09046	100.0	0.459333e-9

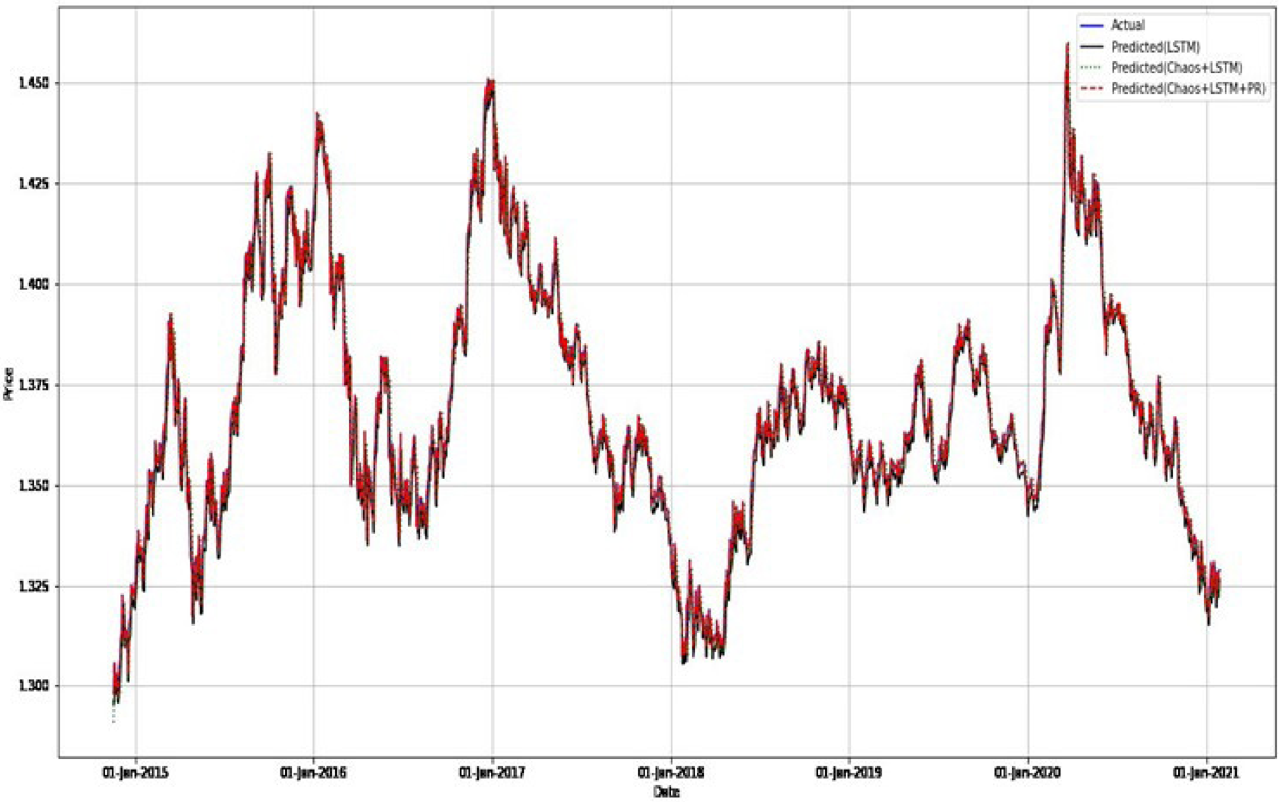

Figure 10. Prediction of SGD/USD test set using the hybrid model.

Table 16. Results of comparison tests for all datasets

Dataset	Chaos+LSTM+PR Vs							
	ARIMA	Prophet	LSTM	CART	RF	Chaos+ CART	Chaos+ RF	Chaos+ LSTM
Crude Oil Price	16.608861	59.504483	1.934142	2.997201	1.975113	2.939101	1.975341	4.139937
Gold Price	25.251906	53.150325	16.150901	11.824691	10.290964	11.871160	10.342319	13.885742
Soybeans Price	49.468927	104.816448	16.430600	18.716957	20.766008	19.101758	20.2501394	22.947899
Nifty 50	31.638381	19.525087	11.207571	28.112393	27.243968	28.109898	27.363947	32.640214
Shanghai Composite Index	46.021043	69.048447	13.989504	16.249247	13.341778	15.619439	12.931739	9.054099
S&P 500	26.782925	25.526353	15.365679	30.816682	30.974349	30.817048	31.015245	13.790424
INR/USD	35.165456	41.220187	17.238165	28.135707	26.598149	28.570410	26.594771	23.306014
JPY/USD	24.014181	27.278746	14.942023	18.165715	15.614305	18.492006	15.929062	14.942097
SGD/USD	64.858459	101.946745	22.672347	25.670308	22.173350	24.298166	21.859957	20.562666

9. Conclusion

A new hybrid model called Chaos+LSTM+PR was introduced in this research paper in order to address the predictive problem of financial time series. First, the presence of chaos in this hybrid is examined by the FTS. The theory of chaos can then be used to model time series chaos. The modeled time series is fed into the LSTM to obtain initial predictions. The error series obtained from LSTM predictions is fitted by PR to obtain error predictions. The error predictions and initial predictions from LSTM are combined to obtain final predictions. Three types of FTS are used to assess the efficiency of the proposed hybrid: foreign exchange rates, commodity prices, and stock market indices. In terms of MSE, MAPE, Dstat, and Theil's U, the suggested hybrid outperforms ARIMA, Prophet, LSTM, CART, RF, Chaos+CART, Chaos+RF, and Chaos+LSTM. A variety of financial and non-financial times can be applied to the proposed hybrid

Acknowledgements

This work is carried out in the research center of Bharathidasan University, Trichy, Tamilnadu.

Conflict of interest

The authors declare no conflict of interest.

References

1. Yao J, Tan CL. A case study on using neural networks to perform technical forecasting of forex. Neurocomputing 2000; 34(1–4): 79–98. doi: 10.1016/S0925-2312(00)00300-3.
2. Dhanya C, Kumar DN. Nonlinear ensemble prediction of chaotic daily rainfall. Advances in Water Resources 2010; 33(3): 327–347. doi: 10.1016/j.advwatres.2010.01.001.
3. Hellstrom T, Holmstrom K. Predicting the stock market. 1998.
4. Magdon-Ismail M, Nicholson A, Abu-Mostafa YS. Financial markets: Very noisy information processing. Proceedings of the IEEE 1998; 86(11): 2184–2195. doi: 10.1109/5.726786.
5. Reid DJ. Combining three estimates of gross domestic product. Economica 1968; 35(140): 431–444. doi: 10.2307/25523.
6. Bates JM, Granger CW. The combination of forecasts. Journal of the Opertional Research Society 1969; 20(4): 451–468. Doi: 10.1057/jors.1969.103.
7. Makridakis S, Andersen A, Carbone R, *et al*. The accuracy of extrapolation (time series) methods: Results of a forecasting competition. Journal of Forecasting 1982; 1(2): 111–153. doi: 10.1002/for.3980010202.
8. Poincaré H. Sur le probléme des trois corps et les équations de la dynamique (French) [On the three-body problem and the equations of dynamics]. Acta Mathematica 1890; 13(1): A3–A270.
9. Williams G. Chaos theory tamed. Washington, D.C.:

CRC Press; 1997.

10. Cavalcante RC, Brasileiro RC, Souza VL, *et al*. Computational intelligence and financial markets: A survey and future directions. Expert Systems with Applications 2016; 55: 194–211. doi: 10.1016/j.eswa.2016.02.006.
11. Bao W, Yue J, Rao Y. A deep learning framework for financial time series using stacked autoencoders and long-short term memory. PloS ONE 2014; 12(7): e0180944. doi: 10.1371/journal.pone.0180944.
12. LeCun Y, Bengio Y, Hinton G. Deep learning. Nature 2015; 521: 436–444. doi: 10.1038/nature14539.
13. Heaton J, Polson NG, Witte JH. Deep learning in finance. arXiv preprint arXiv:1602.06561. 2016. doi: 10.48550/arXiv.1602.06561.
14. Zhang K, Zhong G, Dong J, *et al*. Stock market prediction based on generative adversarial network. Procedia Computer Science 2019; 147: 400–406. doi: 10.1016/j.procs.2019.01.256.
15. Hochreiter S, Schmidhuber J. Long short-term memory. Neural Computation 1997; 9(8): 1735–1780. doi: 10.1162/neco.1997.9.8.1735.
16. Karasu S, Altan A, Bekiros S, Ahmad W. A new forecasting model with wrapper-based feature selection approach using multi-objective optimization technique for chaotic crude oil time series. Energy 2020; 212: 118750. doi: 10.1016/j.energy.2020.118750.
17. Pradeepkumar D, Ravi V. Forex rate prediction using chaos and quantile regression random forest. In: 2016 3rd International Conference on Recent Advances in Information Technology (RAIT); 2016 Mar 3–5; Dhanbad, India. New York: IEEE; 2016. p. 517–522. doi: 10.1109/RAIT.2016.7507954.
18. Huang W, Lai KK, Nakamori Y, Wang S. Forecasting foreign exchange rates with artificial neural networks: A review. International Journal of Information Technology & Decision Making 2004; 3(1): 145–165. doi: 10.1142/S0219622004000969.
19. Pfeiffer M, Hohmann A. Applications of neural networks in training science. Human Movement Science 2012; 31(2): 344–359. doi: 10.1016/j.humov.2010.11.004.
20. Mochón A, Quintana D, Sáez Y, Isasi P. Soft computing techniques applied to finance. Applied Intelligence 2008; 29(2): 111–115. doi: 10.1007/s10489-007-0051-5.
21. Li Y, Ma W. Applications of artificial neural networks in financial economics: A survey. In: 2010 International Symposium on Computational Intelligence and Design; 2010 Oct 29–31; Hangzhou, China. New York: IEEE; 2011. p. 211–214. doi: 10.1109/ISCID.2010.70.
22. Bahrammirzaee A. A comparative survey of artificial intelligence applications in finance: Artificial neural networks, expert system and hybrid intelligent systems. Neural Computing and Applications 2010; 19(8): 1165–1195. doi: 10.1007/s00521-010-0362-z.
23. Pradeepkumar D, Ravi V. Soft computing hybrids for forex rate prediction: A comprehensive review. Computers & Operations Research 2018; 99: 262–284. doi: 10.1016/j.cor.2018.05.020.
24. Durairaj M, Mohan BK. A review of two decades of deep learning hybrids for financial time series prediction. International Journal on Emerging Technologies 2019; 10(3): 324–331.
25. Kim HY, Won CH. Forecasting the volatility of stock price index: A hybrid model integrating LSTM with multiple GARCH-type models, Expert Systems with Applications 2018; 103: 25–37. doi: 10.1016/j.eswa.2018.03.002.
26. Baek Y, Kim HY. ModAugNet: A new forecasting framework for stock market index value with an overfitting prevention LSTM module and a prediction LSTM module. Expert Systems with Applications 2018; 113: 457–480. doi: 10.1016/j.eswa.2018.07.019.
27. Cao J, Li Z, Li J. Financial time series forecasting model based on CEEMDAN and LSTM. Physica A: Statistical Mechanics and Its Applications 2019; 519: 127–139. doi: 10.1016/j.physa.2018.11.061.
28. Pavlidis N, Tasoulis D, Vrahatis MN. Financial forecasting through unsupervised clustering and evolutionary trained neural networks. In: The 2003 Congress on Evolutionary Computation, 2003. CEC'03; 2003 Dec 8–12; Canberra, ACT, Australia. New York: IEEE; 2004. p. 2314–2321. doi: 10.1109/CEC.2003.1299377.
29. Huang SC, Chuang PJ, Wu CF, Lai HJ. Chaos-based support vector regressions for exchange rate forecasting. Expert Systems with Applications 2010; 37(12): 8590–8598. doi: 10.1016/j.eswa.2010.06.001.
30. Pradeepkumar D, Ravi V. Forex rate prediction using chaos, neural network and particle swarm optimization. In: Tan Y, Shi Y, Coello CAC (editors). Advances in Swarm Intelligence. ICSI 2014. Lecture Notes in Computer Science, vol 8795. Cham: Springer; 2014. p. 363–375. doi: 10.1007/978-3-319-11897-0_42.
31. Pradeepkumar D, Ravi V. Forex rate prediction: A hybrid approach using chaos theory and multivariate adaptive regression splines. In: Satapathy

S, Bhateja V, Udgata S, Pattnaik P (editors). Proceedings of the 5th International Conference on Frontiers in Intelligent Computing: Theory and Applications. Advances in Intelligent Systems and Computing, vol 515. Singapore: Springer; 2017. p. 219–227. doi: 10.1007/978-981-10-3153-3_22.

32. Ravi V, Pradeepkumar D, Deb K. Financial time series prediction using hybrids of chaos theory, multi-layer perceptron and multi-objective evolutionary algorithms. Swarm and Evolutionary Computation 2017; 36: 136–149. doi: 0.1016/j.swevo.2017.05.003.
33. Packard NH, Crutchfield JP, Farmer JD, Shaw RS. Geometry from a time series. Physical Review Letters 1980; 45(9): 712. doi: 10.1103. PhysRevLett.45.712.
34. Takens F. Detecting strange attractors in turbulence. In: Rand D, Young LS (editors). Dynamical systems and turbulence, Warwick 1980. Lecture notes in Mathematics, vol 898. Berlin: Springer; 1981. p. 366–381.
35. Akaike H. A new look at the statistical model identification. IEEE Transactions on Automatic Control 1974; 19(6): 716–723. doi: 10.1109/TAC.1974.1100705.
36. Cao L. Practical method for determining the minimum embedding dimension of a scalar time series. Physica D: Nonlinear Phenomena 1997; 110(1–2): 43–50. doi: 10.1016/S0167-2789(97)00118-8.
37. Makridakis S, Hibon M. ARMA models and the Box–Jenkins methodology. Journal of Forecasting 1997; 16(3): 147–163. doi: 10.1002/(SICI)1099-131X(199705)16:3<147::AID-FOR652>3.0.CO;2-X.
38. Hibon M, Makridakis S. Evaluating accuracy (or error) measures. INSEAD [Google Scholar] 1995.