ORIGINAL RESEARCH ARTICLE

A novel road traffic flow prediction model using hybrid Particle Swarm Optimization (PSO) and Radial Basis Function Neural Network (RBFNN)

Shanhua Zhang[1], Hong Ki An[2,*]

[1] *Department of Digital Equipment, Jiangsu Vocational College of Electronics and Information, Huai'an 223003, China*

[2] *Faculty of Civil Engineering & Technology, Universiti Malaysia Perlis, Arau Perlis 02600, Malaysia*

* Corresponding author: Hong Ki An, hongkian@unimap.edu.my

ABSTRACT

Traffic congestion is a major problem in urban areas, leading to increased travel time, air pollution, and fuel consumption. Road impedance function, which describes the relationship between traffic status and travel time, plays an important role in predicting travel time and managing traffic flow. Traditional methods for estimating road impedance function rely on manual calibration and may have limitations in reflecting the complexity of traffic patterns. To address these challenges, researchers have proposed various machine learning models for predicting travel time and road impedance function. In this paper, a hybrid particle swarm optimization—radial basis function neural network model is proposed for improving the accuracy of the road impedance function. The model takes into consideration various vehicle types and is validated using travel time data collected from a road section in Huai'an City, China. The effectiveness of the proposed model is compared with the traditional road impedance function calibrated by nonlinear regression. The experimental results indicate that the Mean Relative Error (MRE) of PSORBFNN is increased by 3.89% and 6.28% respectively when compared with DPNR training samples and validation samples. When compared with DPPSO training and validation samples, the MRE of PSORBFNN is increased by 2.87% and 3.3% respectively. These findings suggest that the proposed model could guide and assist traffic engineers and practitioners in predicting travel time on road sections with improved accuracy.

Keywords: road impedance function; BPR function; PSO; BP neural network

1. Introduction

Traffic congestion is a major challenge faced by cities around the world, leading to increased travel time, air pollution, and fuel consumption. Accurate prediction of road traffic flow is crucial for efficient traffic management and for reducing traffic congestion. Road impedance function, which describes the relationship between traffic status and travel time, is a key factor in predicting traffic flow. The performance function of the link calculates the travel time which varies depending on the amount of traffic allocated to the link. Thus, how to obtain an accurate impedance function that can reflect the actual road travel time has always been the focus of traffic engineers and scholars[1–3]. Speed or travel time is one of the representative indicators of the operating characteristics of roads or road networks. In general, as traffic volume increases, the traffic resistance, that is, the delay or the travel time increases, and it is the Road Impedance Function (RIF) that shows the relationship between the traffic volume

and traffic resistance.

In recent years, machine learning techniques have been widely used to predict traffic flow and road impedance function. Currently, the Bureau of Public Roads (BPR) function developed by the US Federal Highway Administration (FHWA) is the most widely used to calculate the impedance function[4–6]. It reflects the relationship between the travel time and the traffic flow of the road. The classic BPR, however, has several drawbacks when applied to Chinese road environments. This is because mixed traffic is not common in developed countries such as the US, whereas Chinese roads are comprised of two or more modes resulting in inaccuracy of RIF[7]. Moreover, the model does not consider the influence of various vehicle types on the impedance function. Various machine learning methods have been proposed for predicting road traffic flow, including artificial neural networks (ANNs), support vector machines (SVMs), and decision trees. ANNs have been widely used for traffic flow prediction due to their ability to learn from data and capture complex patterns. However, traditional ANN models may have limitations in reflecting the various vehicle types and the dynamic nature of traffic flow. To address this, researchers have proposed various hybrid models, such as the hybrid ANFIS-PSO model and the hybrid RBFNN-PSO model. Zhao et al.[7] considered the RIF in the case of road congestion, and quantified the influencing factors such as intersection density, bus stop density, non-motor vehicle density and saturation. In addition, they constructed an improved BPR function, and calculated the road traffic impedance function utilizing measured data from different roads. Sheffi[8] pointed out that the classic BPR function cannot describe congested road conditions properly due to its simple concept. Thus, the study of BPR function was carried out using various field data. Skabardonis and Dowling[9] also developed the classic BPR function based on congested and uncongested road conditions in order to increase the accuracy of the function. It was found that the application of different road types can enhance the accuracy of road impedance, while road geometries (grade and length) and traffic signals can affect the RIF at intersections[10]. Muhammad et al.[11] studied the RIF focusing on Q-C value to illustrate the interaction between traffic volume and travel speed in Indonesia's road situations, and Zhang et al.[12] calibrated the RIF using the concepts of travel cost, travel time and traffic volume. Zhang et al.[13] constructed a stochastic model of BPR function to increase travel time accuracy in the transportation network. The relationship between the density distribution function of a road section and the coefficient of capacity was taken into account in the study to enhance route choice and travel time forecasting.

Despite the availability of various machine learning models for predicting road traffic flow, there is still a need for more accurate and efficient models that can consider the various vehicle types and complex traffic patterns. This paper proposes a hybrid PSO-RBFNN model that considers various vehicle types and can accurately predict road traffic flow. The proposed hybrid PSO-RBFNN model consists of two stages. In the first stage, the PSO algorithm is used to optimize the parameters of the RBFNN model. In the second stage, the optimized RBFNN model is used to predict road traffic flow. The proposed model is validated using travel time data collected from Yan'an East Road in Huai'an City, China. A further correction is made to optimize the impedance function by combining the two algorithms. The proposed model will be compared with the traditional model in order to check the accuracy of the impedance function. In this research, Yan'an East Road, Huai'an City, China is selected as the study area. The effectiveness of the proposed model is compared with the traditional road impedance function calibrated by nonlinear regression. Overall, the proposed hybrid PSO-RBFNN model has the potential to improve the accuracy of road traffic flow prediction and contribute to the development of more accurate and efficient traffic management strategies.

2. Theoretical background

Road traffic is an essential part of modern life, but it can also be a significant source of congestion, pollution, and accidents. Improving road traffic requires a comprehensive approach that involves various measures and strategies aimed at enhancing safety, efficiency, and sustainability.

One of the key strategies for improving road traffic is to invest in infrastructure. This includes building and maintaining roads, bridges, tunnels, and other transportation facilities that can accommodate increasing traffic volumes and provide safe and efficient travel. Improving infrastructure can also involve implementing smart traffic management systems that use sensors and other technologies to monitor traffic flow and optimize traffic patterns. Another important aspect of improving road traffic is promoting sustainable transportation options, such as public transit, cycling, and walking. These modes of transportation can help reduce the number of vehicles on the road, decrease congestion, and improve air quality. In addition, encouraging the use of electric or hybrid vehicles can also help to reduce emissions and improve environmental sustainability.

Traditional road traffic impedance function models are mathematical models that are used to describe the relationship between traffic flow and travel time on a particular road network. So far, a number of traditional road traffic impedance function models have been established and developed. Most of them use a model to describe the driving conditions of the road, and calibrate the undetermined parameters in the model via the road information collected from the field survey. The US Highway Administration first conducted an in-depth analysis of expressway travel time, obtaining a large amount of basic data through experiments, and calculated the Bureau of Public Roads (BPR) function using regression analysis[14]. The impedance function model was improved by Spiess[15] in order to address the inaccuracies arising due to the high β value in the BPR function and the travel time change range being too small under low saturation conditions. On the basis of queuing theory, a progressive impedance function is proposed by Davidson[16], that is, the travel time is infinite when road traffic volume reaches the road's capacity. Mori et al.[17] further improved the accuracy of the BPR function for when the impedance parameter is too high and the travel time is almost unchanged under low saturation conditions. These models are based on the assumption that as the number of vehicles on the road increases, travel time increases due to congestion.

The most common types of traditional road traffic impedance function models are:

BPR function: The Bureau of Public Roads (BPR) function is a widely used model that was developed in the 1960s[18]. It assumes that the travel time on a road network increases exponentially as the traffic volume increases. The BPR function is expressed as:

$$T = T_o \left(1 + alpha \left(\frac{V}{C} \right)^{beta} \right) \tag{1}$$

where T is the travel time, T_o is the free-flow travel time, V is the traffic volume, C is the capacity of the road, and alpha and beta are parameters that depend on the road characteristics.

Greenshields model: The Greenshields model was developed in the 1930s and is based on the observation that traffic flow on a road network can be approximated as a parabolic function. The Greenshields model is expressed as:

$$V = k(C - Y) \tag{2}$$

where V is the traffic volume, k is a constant, C is the capacity of the road, and Y is the traffic density. The Greenshields model assumes that the travel time is proportional to the inverse of the speed, which is a function of the traffic density.

Underwood model: The Underwood model is a more recent model that was developed in the 1990s. It is based on the observation that traffic flow can be approximated as a logistic function. The Underwood model is expressed as:

$$V = C / (1 + exp(a + b(T - T_O))) \tag{3}$$

where V is the traffic volume, C is the capacity of the road, T is the travel time, T_O is the free-flow travel time, and a and b are parameters that depend on the road characteristics.

3. Proposed model

3.1. Objective function

This paper proposes a new impedance function based on the traditional BPR function as:

$$t = t_0\left(1 + a_1(Q_{tot}/C)^{\beta_1}\right)\left(1 + a_2(Q_{med}/C)^{\beta_2}\right)\left(1 + a_3(Q_{lar}/C)^{\beta_3}\right) \tag{4}$$

where Q_{tot} represents the total traffic volume on the road section within one-hour; Q_{lar} represents the large vehicle traffic volume on the road segment within one-hour; Q_{med} represents the medium-sized vehicle traffic volume on the road segment within one-hour; C represents the traffic capacity on the road segment within one-hour; $a_1, a_2, a_3, \beta_1, \beta_2, \beta_3$ are parameters to be determined. The relationship of Q_{tot} with Q_{med} and Q_{lar} is shown,

$$Q_{tot} = Q_{sma} + \rho \cdot Q_{med} + \mu \cdot Q_{lar} \tag{5}$$

where Q_{sma} represents the traffic volume of passenger cars on the road section within one-hour; ρ represents the coefficient of converting medium-sized cars into passenger cars, which is taken as 1.5 here; μ represents the coefficient of converting large cars into passenger cars, which is taken as 2 here. The obtained traffic volume in 5-minute intervals can be converted to one-hour traffic volume as follows:

$$Q_E = 12 \cdot q_r \tag{6}$$

where q_r represents the five-minute traffic volume from the survey; Q_E represents the traffic volume expanded to one-hour.

According to the above investigation method, N time periods are obtained. Each sample includes the traffic volume of each vehicle type and the travel time under this road condition. The value of each parameter is calibrated by solving the minimum value as follows:

$$y = \frac{\min \sum_{j=1}^{N} \left| t_j - t_0\left(1 + a_1(Q_{j\,tot}/C)^{\beta_1}\right)\left(1 + a_2(Q_{j\,med}/C)^{\beta_2}\right)\left(1 + a_3(Q_{j\,lar}/C)^{\beta_3}\right) \right|}{t_j \cdot N} \tag{7}$$

$$0 < a_1, a_2, a_3 \leq 5 \tag{8}$$

$$0 < \beta_1, \beta_2, \beta_3 \leq 10 \tag{9}$$

where j represents the j-th experimental time period, N is total time periods; $Q_{j\,tot}$ represents the total traffic volume of the j-th time period; $Q_{j\,med}$ represents the traffic volume of the j-th time period of medium-sized vehicles; $Q_{j\,lar}$ represents the j-th time period of large vehicles; t_j represents the actual observed travel time of the j-th time period; $t_0\left(1 + a_1(Q_{j\,tot}/C)^{\beta_1}\right)\left(1 + a_2(Q_{j\,med}/C)^{\beta_2}\right)\left(1 + a_3(Q_{j\,lar}/C)^{\beta_3}\right)$ represents the predicted travel time of the j-th time period; $\left| t_j - t_0\left(1 + a_1(Q_{j\,tot}/C)^{\beta_1}\right)\left(1 + a_2(Q_{j\,med}/C)^{\beta_2}\right)\left(1 + a_3(Q_{j\,lar}/C)^{\beta_3}\right) \right|/t_j$ is the relative error of the j-th time period, and y is the objective function to represent the mean relative error (MRE).

In order to further improve the accuracy of the impedance function, the parameter z is determined by

$$t_j = z_j \cdot \hat{t}_j \tag{10}$$

where t_j represents the actual travel time of the j-th time period; $\hat{t}_j$ is the travel time obtained by Equation (4), which represents the travel time of the j-th time period based on the DPPSO calibration impedance function; z_j is the correction parameter of the j-th time period. Therefore, the j-th time period z_j, the correction parameter of the time periods, is equal to the ratio of the actual transit time of the j-th time period to the predicted travel time t_j of the j-th time period.

3.2. Hybrid PSORBFNN

The DPPSO calibration impedance function based on RBFNN is called PSORBFNN. Although the calibrated impedance function based on DPPSO can greatly improve its accuracy, there is still a large

difference between the travel time of the road section calculated by optimizing the impedance function and the travel time obtained by the actual investigation.

The hybrid PSO-RBFNN model is a combination of the Particle Swarm Optimization (PSO) algorithm and the Radial Basis Function Neural Network (RBFNN) that is used to predict road traffic flow. The model consists of three main components: data preprocessing, PSO-RBFNN training, and traffic flow prediction.

Data preprocessing: In this stage, the raw traffic data is preprocessed to remove outliers, missing values, and other noise. The preprocessed data is then normalized to ensure that all the input features have a similar scale. The normalized data is then partitioned into training and testing sets.

PSO-RBFNN training: In this stage, the PSO algorithm is used to optimize the weights and biases of the RBFNN. The PSO algorithm searches for the optimal set of weights and biases that minimize the prediction error of the RBFNN. The RBFNN is trained on the preprocessed and normalized training data. The architecture of the RBFNN typically consists of an input layer, a hidden layer, and an output layer. The input layer receives the normalized input features, while the output layer produces the predicted traffic flow. The hidden layer consists of RBF neurons that use radial basis functions to transform the input features into a more suitable representation for prediction.

Traffic flow prediction: In this stage, the trained PSO-RBFNN model is used to predict the traffic flow for a given time period. The input features to the model include historical traffic flow data, time of day, and other relevant factors such as weather and special events. The predicted traffic flow is then compared with the actual traffic flow to evaluate the accuracy of the model.

3.3. Model validation

The calibration impedance function can be obtained by calculating the deviation between the predicted value and the actual value of the travel time, and the MRE can be expressed as

$$MRE = \frac{\sum_{j=1}^{N}|t_j - \hat{t}_j|}{t_j \cdot N} \tag{11}$$

where j represents the j-th experimental time period, N is total time periods ($N = 144$); t_j represents actual travel time of j-th time period; $\hat{t}_j$ represents predicted travel time of j-th time period.

4. Result and discussion

4.1. Data collection

In order to collect traffic volume and travel time, Yan'an East Road, Qingjiangpu District, Huai'an City was selected. The survey was conducted from 7:00 to 19:00 on 13 March 2019. The road is two-way with two lanes, no central divider in the middle, and no divider between non-motor vehicles and motor vehicles. The width of each lane is three meters, and the total length of the survey area is around 810 meters.

The time difference between the two cameras was used to obtain the travel time of the vehicle. At the same flow rate, there is no significant difference in the appearance of average, highest and lowest speed and density. In addition, shorter time periods allow for more data acquisition time periods, therefore, this study set five-minute time intervals in which to collect data (assuming that the traffic volume on the road remains unchanged within those five minutes).

In order to calibrate the value of each parameter in the RIF, the interval observation method was used. A camera was installed at each end of the survey area, on a section of road with no intervening intersections, as shown in **Figure 1**. The number of vehicles captured by the two cameras was the same when the vehicle under investigation did not turn around halfway. Moreover, the traffic volume of each vehicle type can be obtained by counting the captured results of any one camera, simultaneously using the license plate to

identify the vehicle. When the vehicles were captured by camera 1 and then later by camera 2, the time difference between the two cameras was recorded to obtain the travel time of the vehicle. Furthermore, vehicles could be categorized into three types in accordance with their passenger and goods capacity as shown in **Table 1**.
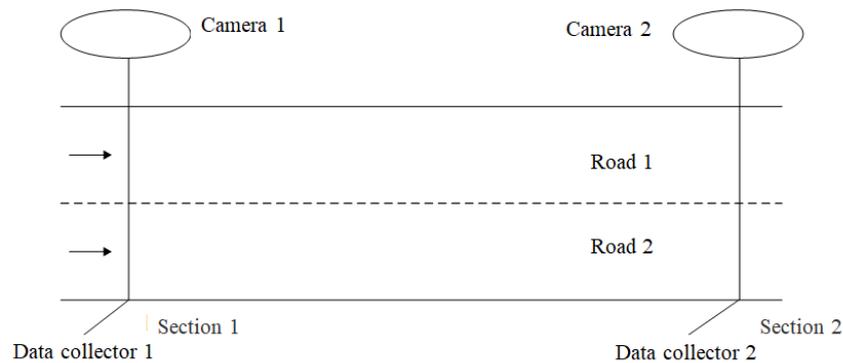
Figure 1. Experimental diagram of interval observation method.

Table 1. Classification and conversion factor of vehicle types.

Vehicle type classification	Passenger Car Unit (PCU) value	Description
Small car	1.0	The passenger capacity of the bus is less than or equal to 19 people. The cargo capacity of the truck is less than or equal to 2 tons.
Medium car	1.5	The passenger capacity of the bus is greater than 19 people. Trucks carrying between 2 and 7 tons.
Large car	2.0	Trucks carrying between 7 and 14 tons.

Figure 2 illustrates traffic volume of each type in 5-minute intervals and small, medium and large cars are mixed on the study area. Also, there were four peak periods during morning, afternoon and evening. The proportion of small cars was higher than of the other two vehicle types and the maximum volume was 41 pcu from 18:10–18:15. From the data survey, it can be found that travel time and road section capacity are 63 seconds and 750 pcu/h respectively.

With respect to travel time, there is a proportional relationship between traffic volume and travel time. During four peak periods (8:30–9:00, 12:00–12:30, 13:35–13:50 and 17:30–18:00) travel time increased with the increase of traffic volume.
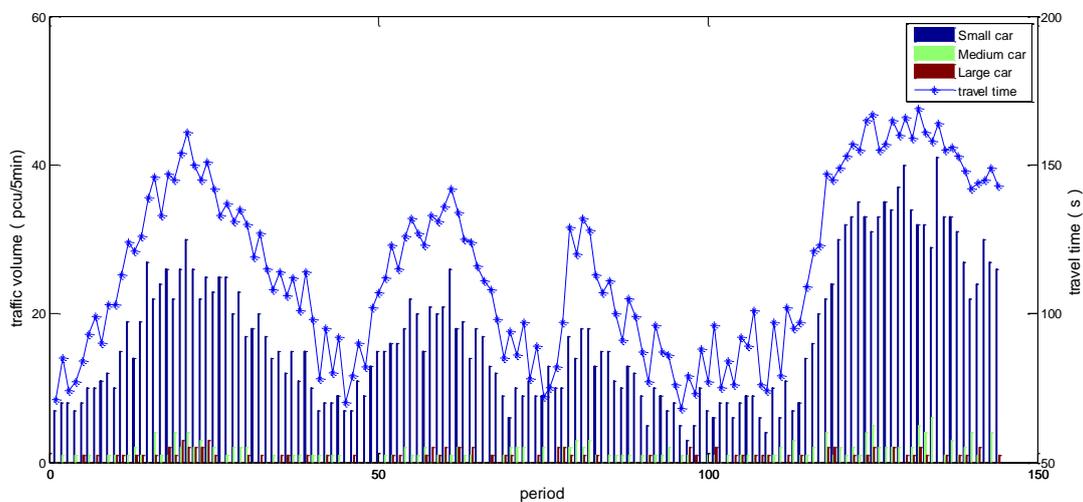
Figure 2. Traffic volume of each vehicle type and travel time survey.

4.2. Verification of RIF optimized by PSO algorithm

Bureau of Public Roads[19] stated that the traditional impedance function determines the parameters based on nonlinear regression (DPNR). The nonlinear function Equation (1) is converted into linear function, and the parameters to be determined are calibrated by using the least square method in linear regression, and the scattered points and linear straight lines can be obtained. The scattered points are approximately linear and evenly distributed on both sides of the regression line as shown in **Figure 3a**. The optimized RIF determines the parameters based on PSO algorithm (DPPSO). The Equation (5) is used as the fitness function, and the PSO algorithm is used to calibrate the parameters to be determined. As shown in **Figure 3b**, the MRE is gradually converging by using PSO algorithm to search the undetermined parameters. When the number of iterations reaches the 20th, the algorithm converges stably, and the MRE convergence value is 4.53%.

(a) Regression scatter plot   (b) Convergence diagram
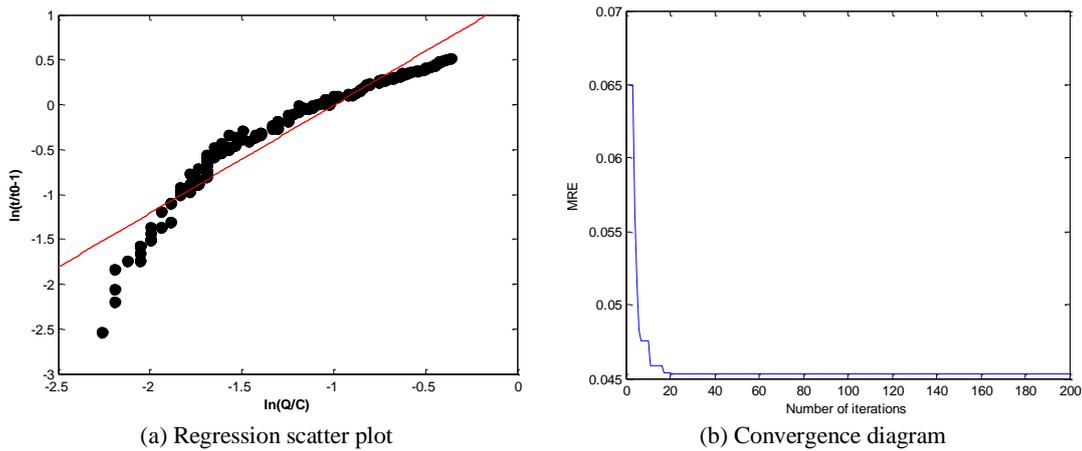
Figure 3. Parameter calibration based on DPNR and DPPSO.

The predicted travel time of DPNR by Equation (1) with parameter value ($\alpha = 3.3$ and $\beta = 1.2$) and DPPSO by Equation (2) with parameter values ($a_1 = 2.5$, $a_2 = 2.53$, $a_3 = 0.16$, $\beta_1 = 0.9$, $\beta_2 = 8.69$ and $\beta_3 = 5.86$) can be illustrated in **Figure 4**. DPNR and DPPSO have high prediction accuracy for travel time in most time periods. In the morning peak and evening peak, however, the predicted travel time increases sharply compared with field data due to the large traffic volume.
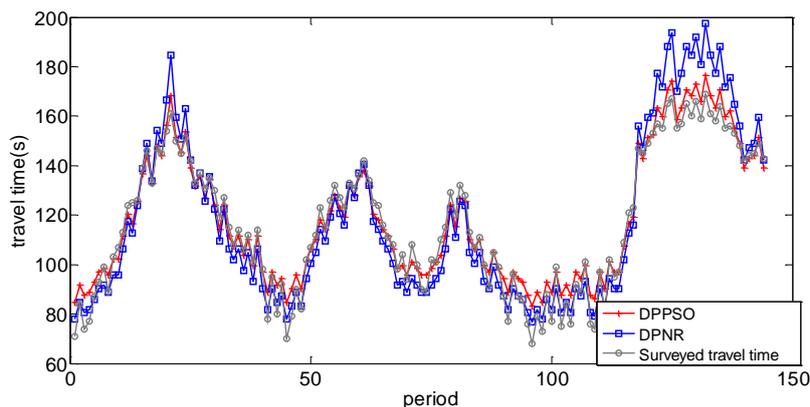
Figure 4. Travel time comparison.

According to **Table 2**, it can be seen that the impedance function based on DPPSO matched well with the field data with the MRE value of 4.53%, whereas DPNR has a MRE value of 6.15%. This means DPPSO is more suitable as a function of road impedance.

7

Table 2. MRE of travel time based on DPNR and DPPSO.

Parameter calibration method	DPNR	DPPSO
MRE (%)	6.15	4.53

4.3. Verification of RIF correction based on DPPSO

The transit time of the road segment calculated based on the impedance function calibrated by DPPSO is used as the input element of the RBFNN, and the correction parameters are used as the output element of the RBFNN. The total number of time periods is 144, of which the first 96 time periods are training samples to train the RBFNN to test the fitting error of the impedance function; the last 48 time periods are the testing samples to test the prediction error of the impedance function. The number of times of network training is 30,000 times and learning rate is 0.05. The range of hidden layers in this paper is 3–12. In order to determine the optimal number of hidden layers, experiments are conducted with different hidden layers, and the MRE of training samples and testing samples in accordance with the number of hidden layers are shown in **Table 3**.

Table 3. MRE of training samples and testing samples under different hidden layers.

No. of hidden layers	MRE of training samples (%)	MRE of testing samples (%)
3	1.45	1.71
4	1.44	1.67
5	1.47	1.69
6	1.45	1.72
7	1.47	1.78
8	1.50	1.69
9	1.51	1.76
10	1.52	1.69
11	1.57	1.79
12	1.55	1.78

The MRE between training samples and test samples is minimal (1.44%, 1.67%) when the number of hidden layer nodes is four, hence this paper chooses four as the number of hidden layer nodes for the RBFNN.

The correction coefficient z_j of 144 samples can be obtained based on RBFNN. The corrected road travel time can be obtained by multiplying the correction coefficient by the prediction result based on DPPSO calibration impedance function. **Figure 5** shows the value of training samples correction coefficient z_j. The corrected parameter values of the training samples are concentrated in the interval [0.8–1.05], and most of the values are around 1, so the adjustment amount of the training samples is small.
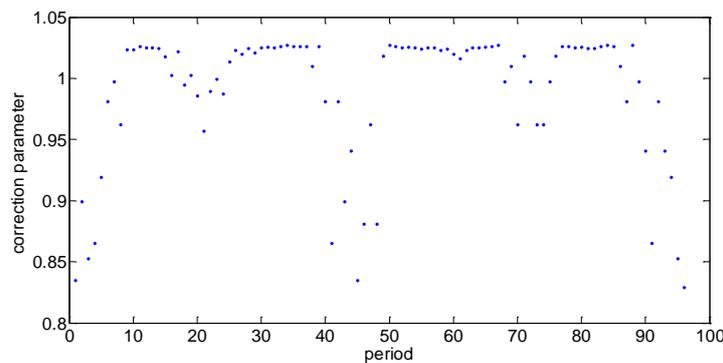
Figure 5. Value of training samples correction coefficient z_j.

8

Figure 6 shows the comparison of travel time after the training samples correction between PSORBFNN, DPPSO, DPNR and survey data. The values of MRE were 5.33% (DPNR) and 4.31% (DPPSO), whereas the MRE value was reduced to 1.44% by PSORBFNN. Thus, it can be seen that PSORBFNN matches the survey data well and it is able to improve the model accuracy.
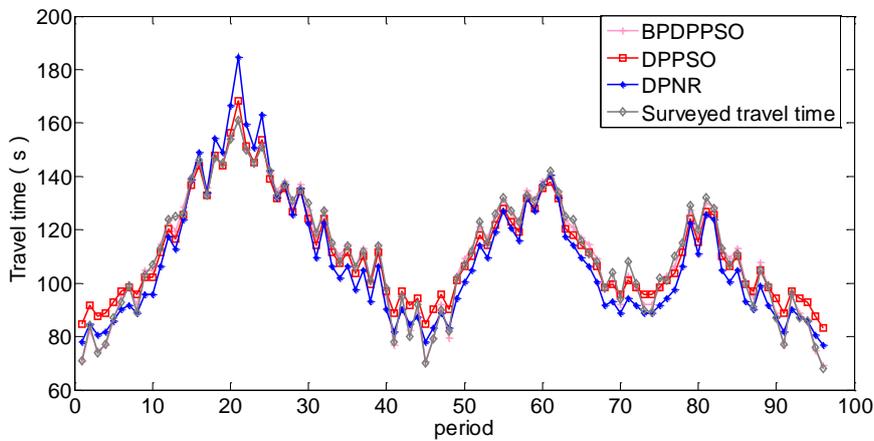
Figure 6. Travel time comparison between PSORBFNN, DPPSO and DPNR in the training sample time periods.

Figure 7 shows the value of testing samples correction coefficient z_j. Also, it can be seen that the corrected parameter values of the testing samples are concentrated in the interval [0.8−1.05]. By contrast with training samples, the concentration degree of scattered points is low, and the adjustment amount of verification samples is a little large.
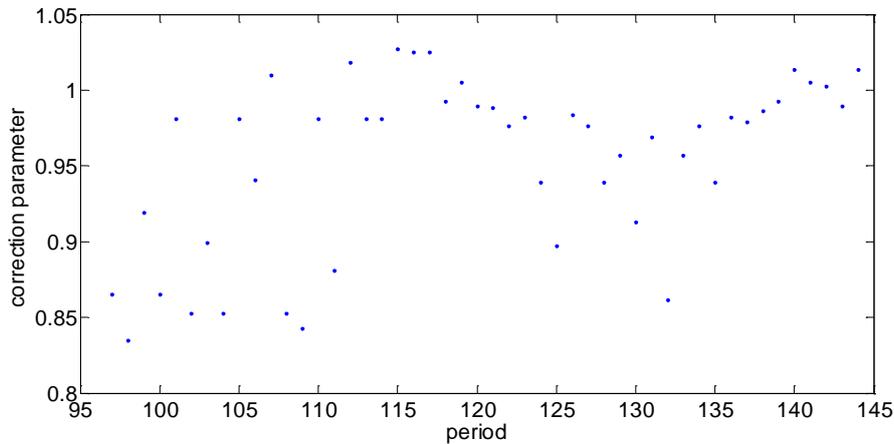
Figure 7. Value of testing samples correction coefficient z_j.

Figure 8 shows the predicted value of the travel time after the testing samples correction. In terms of MRE assessment, errors of 7.95%, 4.97% and 1.67% were achieved for DPNR, DPPSO and PSORBFNN respectively. It can be seen that the prediction accuracy of PSORBFNN is higher than that of the other two methods, which verifies the effectiveness of this method.

9
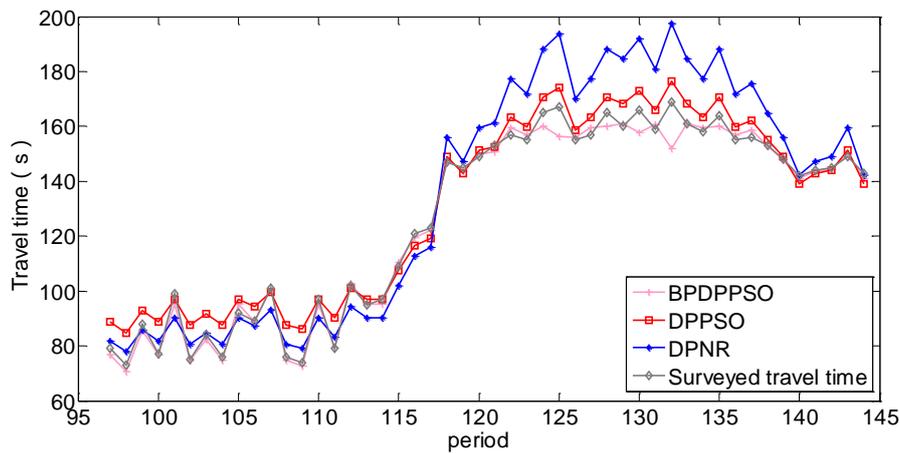
Figure 8. Travel time comparison between PSORBFNN, DPPSO and DPNR in the testing sample time periods.

5. Conclusion

This paper proposed a novel road traffic flow prediction model that combines the strengths of Particle Swarm Optimization (PSO) and Radial Basis Function Neural Network (RBFNN) techniques. The proposed hybrid PSO-RBFNN model addresses the limitations of traditional traffic flow prediction models in accurately capturing the complex nonlinear relationships between traffic flow and other relevant factors such as weather and special events. The model was evaluated on real-world traffic data and compared with other state-of-the-art traffic flow prediction models. The results showed that the hybrid PSO-RBFNN model outperformed these models in terms of accuracy and robustness. The model was able to accurately predict traffic flow with high precision, and it was able to handle the noise and outliers in the data effectively. Furthermore, the proposed model can be used to support various transportation-related applications such as intelligent transportation systems (ITS), traffic management, and urban planning. The accuracy and efficiency of the model can help reduce traffic congestion, improve road safety, and promote more sustainable transportation systems.

Author contributions

Conceptualization, SZ; methodology, SZ; software, SZ; validation, SZ and HKA; formal analysis, SZ and HKA; investigation, SZ and HKA; resources, SZ; data curation, SZ; writing—original draft preparation, SZ; writing—review and editing, HKA; visualization, SZ; supervision, HKA; project administration, SZ; funding acquisition, SZ.

Funding

Conflict of interest

The authors declare no conflict of interest.

References

1. He N, Zhao S. Discussion on influencing factors of free-flow travel time in road traffic impedance function. *Procedia-Social and Behavioral Sciences* 2013; 96: 90–97. doi: 10.1016/j.sbspro.2013.08.013
2. Tan H, Yang Y, Zhang L. Improved BPR function to counter road impedance through OD matrix estimation of freight transportation. *Journal of Highway and Transportation Research and Development (English Edition)* 2017;

11(2): 97–102. doi: 10.1061/JHTRCQ.0000572

3. Huo J, Wu X, Lyu C, et al. Quantify the road link performance and capacity using deep learning models. *IEEE Transactions on Intelligent Transportation Systems* 2022; 23(10): 18581–18591.

4. Kim YG, Mahmassani HS. *Link Performance Functions for Urban Freeways with Asymmetric Car-truck Interactions*. Transportation Research Record; 1987. pp. 32–39.

5. He N, Zhao S. Urban road traffic impedance function—Dalian city case study. *Journal of Highway and Transportation Research and Development* 2014; 8(3): 90–95. doi: 10.1061/JHTRCQ.0000402

6. Liang L, Yang Y, Wang H, et al. Traffic impedance estimation driven by trajectories for urban roads. In: Proceedings of the 3rd International Conference on Vision, Image and Signal Processing; 26–28 August 2019; Vancouver BC, Canada. pp. 1–7.

7. Zhao F, Fu L, Zhong M, et al. Development and validation of improved impedance functions for roads with mixed traffic using Taxi GPS trajectory data and simulation. *Journal of Advanced Transportation* 2020; 2020: 7523423. doi: 10.1155/2020/7523423

8. Sheffi Y. *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*. Prentice-Hall; 1985.

9. Skabardonis A, Dowling R. Improved speed-flow relationships for planning applications. *Transportation Research Record: Journal of the Transportation Research Board* 1997; 1572(1): 18–23. doi: 10.3141/1572-03

10. Dong J, Shen G. A weight-based road impedance function model. *Advanced Materials Research* 2013; 756–759: 2750–2755. doi: 10.4028/www.scientific.net/AMR.756-759.2750

11. Muhammad ZI, Sumi T, Munawar A. Implementation of the 1997 Indonesian highway capacity manual (MKJI) volume delay function. *Journal of the Eastern Asia Society for Transportation Studies* 2010; 8(2): 122–123. doi: 10.11175/easts.8.350

12. Zhang C, Qin J, Zhang M, et al. Practical road-resistance functions for expressway work zones in occupied lane conditions. *Sustainability* 2019; 11(2): 382. doi: 10.3390/su11020382

13. Zhang J, Liu M, Zhou B. Analytical model for travel time-based BPR function with demand fluctuation and capacity degradation. *Mathematical Problems in Engineering* 2019; 2019: 5916479. doi: 10.1155/2019/5916479

14. TRB. *Highway Capacity Manual*. Transportation Research Board, National Research Council; 2010.

15. Spiess H. Conical volume-delay functions. *Transportation Science* 1990; 24(2): 153–158. doi: 10.1287/trsc.24.2.153

16. Davidson KB. The theoretical basis of a flow-travel time relationship for use in transportation planning. *Australian Road Research* 1978; 8(1): 32–35.

17. Mori U, Mendiburu A, Álvarez M, Lozano JA. A review of travel time estimation and forecasting for advanced traveler information systems. *Transportmetrica A: Transport Science* 2015; 11(2): 119–157. doi: 10.1080/23249935.2014.932469

18. Zhang Q, Zhu Y, Wang Z, et al. Reliability assessment of distribution network and electric vehicle considering Quasi-Dynamic traffic flow and vehicle-to-grid. *IEEE Access* 2019; 7: 131201–131213. doi: 10.1109/ACCESS.2019.2940294

19. Bureau of Public Roads. *Traffic Assignment Manual for Application with a Large Speed Computer*. Urban Planning Division, US Department of Commerce; 1964.